A STUDY OF AN AUTOMATIC POWER COMPENSATOR WITH AIRSPEED AND ANGLE OF ATTACK AS CONTROLLING INPUTS

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AIRSPEED AND ANGLE OF ATTACK AS CONTROLLING INPUTS

by

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ABSTRACT

A study of an Automatic Power Compensator with inputs of airspeed and angle of attack was conducted by means of a simplified analytical analysis and analogue simulation. Digital Computer programming was used for data reduction. The Chance Vought F-8 aircraft was used as the vehicle for the study. Loop gains were initially set by means of a simplified analysis based on the independent phugoid and short period motions. The validity of such an analysis was demonstrated by analogue simulation of the complete system. The system is feasible both for independent use or as a component of a complete automatic flight system.

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TABLE OF SYMBOLS

(For those symbols not defined in the text)

ac	Aerodynamic center
B	(I_{yy}) Moment of Inertia about the Y axis
c	chord (MAC of wing)
C_L	coefficient of Lift
C_D	coefficient of Drag
C_m	coefficient of Moment
cg	center of gravity
g	acceleration of gravity
K_α	angle of attack proportional gain
K_u	airspeed proportional gain
$k_u K_u$	airspeed integral gain
L	Lift force
l_t, ℓ_t	tail length - distance from cg to ac of horizontal tail
m	mass
n	vertical acceleration perturbation (g units)
u	horizontal velocity perturbation
U_o	steady state velocity
V	reference velocity ($V \triangleq U_o$)
w	vertical velocity perturbation
W	aircraft weight
S_w	reference area (wing area)

S	Laplace operator
T	thrust
(t)	indicates variable in time domain
α	angle of attack, $\frac{u}{U_o}$ (see Fig 2)
η	elevator deflection (see Fig 2)
θ	angle above horizontal (see Fig 2)
γ	flight path angle (see Fig 2)
ρ	atmospheric density
τ	time constant for delays (see Table IV)
$\left\vert \quad \right\vert$	indicates determinant expansion
$\left(\quad \right)$	indicates matrix formulation

Superscripts:

$\dot{}$	$\frac{d(\quad)}{dt}$
$\ddot{}$	$\frac{d^2(\quad)}{dt^2}$

Subscripts:

o	initial or steady state value
e	error quantity or signal
c	command signal
f	delayed signal

Aerodynamic derivatives

The standard NACA notation is used for the non-dimensional derivatives in that

$$C_{Lu} = \frac{\partial C_L}{\partial u} \; ; \; C_{m_\alpha} = \frac{\partial C_m}{\partial \alpha} \; ; \text{ etc. (See Table II)}$$

1. Introduction.

This study is a continuation of a series of investigations designed to evaluate various types of "Auto-throttle" devices. The study was instituted in the Aeronautical Engineering Department of the United States Naval Postgraduate School in 1963. [8, 9]

The need for such a device on high performance carrier aircraft and the basic concepts relative to the system have been previously discussed. Briefly, the power compensator is a device meant to automatically stabilize the airspeed at a pre-selected value without attention from the pilot. The pilot then maintains the desired attitude or glide slope. This concept is not limited to, but is especially desirable during carrier type approaches where modern, high performance, swept-wing aircraft must operate in a range of inherent speed instability. With the heavier aircraft now being programmed for carrier use lower landing speeds are most desirable. By allowing the pilot to focus his attention on only one correction during approach, a power compensator permits the safe utilization of slower approach speeds. Basically then, the automatic power compensation is a means of artificially tailoring the aircrafts inherent flight characteristics in the low speed region to meet the opposing requirements of safety and lower landing speeds. In this sense such a device could prove applicable, even necessary, to the landing of the larger super-sonic transport aircraft now under development.

The initial study demonstrated the need for such a device by showing real time, pilot controlled, analogue simulated mirror approaches with and without an auto-throttle. [8] The second paper attempted to improve the systems by using different inputs. [9] The systems analyzed thus far use inputs of airspeed/pitch angle and angle of attack/normal acceleration. These are systems of the type investigated by the United States Navy for operational use. The angle of attack/normal acceleration system is presently in use on operational aircraft.

Utilizing the results of the previous studies, Professor E. J. Andrews of the U. S. Naval Postgraduate School proposed a much simplified analytic means of setting loop gains. [4]

This paper will investigate a power compensator that uses inputs of airspeed and angle of attack. Within the knowledge of the author no such system is in use at present. This type of a system might be thought of as a "hybrid system" since it utilizes one of the inputs from each of the two systems investigated thus far. The motivation for such a system lies in the fact that airspeed and angle of attack are two of the most "basic" governing parameters of flight. Since these parameters are related by

$$(1 + n)W = \frac{1}{2} \rho V^2 S C_{L\alpha} \alpha$$

the values of airspeed and angle of attack will be related to the aircraft weight and the desired type of flight path. For a given aircraft weight, the longitudinal flight path is determined completely by the elevator position and the throttle setting (airspeed).

Therefore, with such an auto-throttle stablizing airspeed, the pilot (or autopilot) need only control the pitch attitude to obtain any desired flight path. This type of an auto-throttle would be especially useful for flying a pre-determined flight path designed to give maximum range, maximum rate of climb or a desired glide path over a range of airspeeds.

In the investigation of this type of a power compensator, the simplified analytic approach will be demonstrated in approximating the desired gain settings. The Chance-Vought F-8 aircraft was used as the vehicle in the previous studies and will be used in this study.

The equations of motion are solved using an analogue computer. A digital computer is used to determine aircraft parameters and roots of characteristic equations.

All aerodynamic parameters are used in the English dimensional form. All equations and programming are given in real time.

The aerodynamic data used was obtained from the Chance Vought Division of Ling-Temco Vought and an earlier report on an evaluation of a power compensator. [6, 7]

A word of appreciation is due Professor E. J. Andrews for his assistance during this investigation.

2. Discussion.

2.1 General

The system under study is actually equivalent to the angle of attack/normal acceleration system evaluated previously. The auto-throttle equation for the α/n system is:

Eq(2-1) $$T_c = K_1\alpha - K_2 n$$

Since $L = \frac{1}{2}\rho(V + u)^2 S C_L = \frac{1}{2}\rho S C_L(V^2 + 2uV + u^2)$

Then $\frac{\partial L}{\partial \alpha} = \frac{1}{2}\rho V^2 S C_{L\alpha}$; $\frac{\partial L}{\partial u} = \frac{1}{2}\rho S C_L(2V + 2u) \triangleq S C_L V \rho$

$$nW = \Delta L$$

$$\frac{\partial n}{\partial \alpha} = \frac{1}{W}\frac{\partial L}{\partial \alpha} \; ; \; \frac{\partial n}{\partial u} = \frac{1}{W}\frac{\partial L}{\partial u}$$

Therefore:

$$n = \frac{1}{2}\rho V^2 \frac{S}{W} C_{L\alpha}\alpha + \rho\frac{S}{W}C_L V u = K_3\alpha - K_4 u$$

The Auto-throttle equation becomes:

$$T_c = K_1\alpha - K_2(K_3\alpha + K_4 u) = (K_1 - K_2K_3)\alpha - K_2K_4 u$$

Eq(2-2) $T_c = K_\alpha \alpha - K_u u$

A block diagram of the complete system is shown in Figure 1.

2.2 Airframe Equations

The 3-degree of freedom, small perturbation, aircraft longitudinal equations were used to solve for the airframe motion. The equations shown below are as developed in any standard text on aerodynamic stability. [3] The definitions of the derivatives are listed in Table I. The dimensional derivatives were computed from the non-dimensional derivatives utilizing a digital computer program given in the Appendix. The equations used for this conversion are given in Table II. The values of the dimensional derivatives obtained are listed in Table III along with all other pertinent flight parameters.

Eqs(2-3) $\dot{u} = X_u u + X_w w + X_q\dot{\theta} - g\cos\theta_o\,\theta + X_T T + X_\eta \eta$

$$(1 - Z_{\dot{w}})\dot{w} = Z_u u + Z_w w + (Z_q + U_o)\dot{\theta} - g\sin\theta_o\,\theta + Z_T T + Z_\eta\eta$$

$$\ddot{\theta} = M_u u + M_w w + M_{\dot{w}}\dot{w} + M_q\dot{\theta} + M_T T + M_\eta \eta$$

The angular and velocity relationships are defined in Figure 2. The derivatives concerning thrust are dervied in Figure 3. These equations were reduced by inserting the values

$$X_q = Z_q = Z_{\dot{w}} = 0 ; \quad \alpha = \frac{w}{U_o}$$

and rearranged to give

Eqs(2-4)

$$\dot{u} = X_u u + U_o X_w \alpha - g\cos\theta_o \theta + X_T T + X_\eta \eta$$

$$\dot{\alpha} = \frac{Z_u}{U_o} u + Z_w \alpha + \dot{\theta} - \frac{g\sin\theta_o}{U_o} \theta + \frac{Z_T}{U_o} T + \frac{Z_\eta}{U_o} \eta$$

$$\ddot{\theta} = M_u u + U_o M_w \alpha + U_o M_{\dot{w}} \dot{\alpha} + M_q \dot{\theta} + M_T T + M_\eta \eta$$

2.3 Power Compensator

The equation of the power compensator is:

Eqs(2-5)

$$T_c = \frac{K_\alpha}{1 + \tau_\alpha S} \alpha_e - \frac{K_u}{1 + \tau_s S} (1 + \frac{k_u}{S}) u_e$$

where τ_α and τ_s are estimates of the angle of attack and airspeed measurement delays respectively. It can be seen from this equation that the power compensator will apply a forward (+) thrust for a nose up (+) angle of attack perturbation and an increase in airspeed (+ u_e) will cause a reduction of thrust. The compensator utilizes angle of attack and airspeed proportional feedbacks plus an airspeed integration to insure zero steady state errors in the airspeed loop.

2.4 Transfer Functions

The transfer functions used are defined in Table IV. The delays were obtained from various sources. [6, 7] The airframe transfer functions are derived in the Appendix from the airframe Laplace equations shown below in matrix form. The airframe transfer functions were evaluated by means of a digital computer program given in the Appendix.

Eqs (2-6)

$$\begin{pmatrix} S - X_u & -U_o X_w & +g\cos\theta_o \\ \dfrac{-Z_u}{U_o} & +(S - Z_w) & -S + \dfrac{g\sin\theta_o}{U_o} \\ -M_{u} & -U_o(M_w + SM_{\dot{w}}) & S^2 - M_q S \end{pmatrix} \begin{pmatrix} u \\ \alpha \\ \theta \end{pmatrix}$$

$$= \begin{pmatrix} X_T & X_\eta \\ \dfrac{Z_T}{U_o} & \dfrac{Z_\eta}{U_o} \\ M_T & M_\eta \end{pmatrix} \begin{pmatrix} T \\ \eta \end{pmatrix}$$

The open and closed loop transfer functions were derived from Figure 1 using standard methods.

Open Loop:

Eqs (2-7)

$$\frac{u}{u_c} = D_u \frac{T_c}{u_c} \frac{u}{T} \frac{\dfrac{T_f}{T_c}\dfrac{T}{T_f}}{1 + \dfrac{T_f}{T_c}\dfrac{T}{T_f} D_u \dfrac{T_c}{\alpha_e}\dfrac{\alpha_e}{T}}$$

Eqs (2-8)

$$\frac{u}{u_c} = \frac{\dfrac{T_c}{u_e}\dfrac{T_f}{T_c}\dfrac{T}{T_f}\dfrac{u}{T}}{1 + D_u \dfrac{T_f}{u_c}\dfrac{T_f}{T_c}\dfrac{T}{T_f}\dfrac{u}{T} - D_\alpha \dfrac{T_c}{\alpha_e}\dfrac{T_f}{T_c}\dfrac{T}{T_f}\dfrac{\alpha_e}{T}}$$

Only the airspeed transfer functions are given above. Transfer functions for α/α_e, u/η, α/η could also be developed easily from the above expression since the denominator, ie. the characteristic equation, would be the same in all cases.

2.5 Digital Computer Programs

The Digital Computer programs used in the previous studies were rewritten in FORTRAN 63 in a subroutine format. These programs are used to convert either wind tunnel data or non-dimensional derivatives to the dimensional derivatives necessary for the Analogue Program. These derivatives are then used as inputs to other subprograms which analize the basic airframe motion and compute the airframe transfer functions for the inputs of elevator and thrust. Thus it is possible to proceed directly from wind tunnel data to the response of the airframe for several flight conditions, using these results to analize any gain changes necessary in the control loops. These programs are contained in the Appendix.

2.6 Complete Analysis

The above transfer functions, both open and closed loop, could now be used to analyze the system. This involves a 5th order characteristic equation with at least three variable gains. The numerators are of third order or less. The characteristic equation is evaluated in the Appendix for later use in a Root Locus Diagram. At this point the methods of Root Locus plots combined with frequency response methods could be used to analyze the response for various combinations of gains to elevator inputs, and step gusts of u and α. This procedure would require an attack by digital computor methods for at least computing the roots of the various polynomials generated for each value of the gains used and then perhaps estimating the response obtained by means of simplified Bode plots or Root Locus plots. But this rigorous, automatic control engineering approach at this point is unnecessary and uneconomical if not also mathematically unsound. Within the accuracy of the assumption inherent in the basic airframe equations (Eqs. 2-3) a much simplified approach is justified. In fact, it is known that the responses calculated by this "exact" method are not only in error, but for the case of shear gusts, can give unconservative results for structural loading parameters. [3]

2.7 Simplified Analysis

This analysis is based on the standard assumption of reducing the airframe equations of motion by assuming two independent motions; one at constant angle of attack, and the other at constant airspeed. Using this assumption, Eqs. (2-6) were separated and reduced as shown below: (negligible values from Table III are omitted)

For Phugoid motion with $\alpha = \frac{g \sin\theta_o}{U_o} = 0$; $\cos\theta_o = 1.0$

Eqs(2-9)
$$\begin{pmatrix} S - X_u & g \\ -\frac{Z_u}{U_o} & -S \end{pmatrix} \begin{pmatrix} u \\ \theta \end{pmatrix} = \begin{pmatrix} X_T & 0 \\ 0 & \frac{Z_\eta}{U_o} \end{pmatrix} \begin{pmatrix} T \\ \eta \end{pmatrix}$$

Then

Eqs(2-10)

$$\frac{u}{T} = \frac{\begin{vmatrix} X_T & g \\ 0 & -S \end{vmatrix}}{\begin{vmatrix} S - X_u & g \\ -\frac{Z_u}{U_o} & -S \end{vmatrix}} = \frac{SX_T}{S^2 - X_uS - g\frac{Z_u}{U_o}}$$

Eqs(2-11)

$$\frac{u}{\eta} = \frac{-g\frac{Z_\eta}{U_o}}{S^2 - X_uS - g\frac{Z_u}{U_o}}$$

For the Short Period Motion with u = 0

Eqs(2-12)

$$\begin{pmatrix} S - Z_w & -S \\ -U_o(M_w + SM_{\dot{w}}) & S^2 - M_qS \end{pmatrix}\begin{pmatrix} \alpha \\ \theta \end{pmatrix} = \begin{pmatrix} 0 & \frac{Z_\eta}{U_o} \\ M_T & M_\eta \end{pmatrix}\begin{pmatrix} T \\ \eta \end{pmatrix}$$

Eqs(2-13)

$$\frac{\alpha}{T} = \frac{\begin{vmatrix} 0 & -S \\ M_T & S^2 - M_qS \end{vmatrix}}{\begin{vmatrix} S - Z_w & -S \\ -U_o(M_w + SM_{\dot{w}}) & S^2 - M_qS \end{vmatrix}}$$

$$\frac{\alpha}{T} = \frac{M_T}{S^2 - (Z_w + M_q + U_oM_{\dot{w}})S + Z_wM_q - U_oM_w}$$

Eqs(2-14)

$$\frac{\alpha}{\eta} = \frac{M_\eta - \frac{M_qZ_\eta}{U_o}}{S^2 - (Z_w + M_q + U_oM_{\dot{w}})S + Z_wM_q - U_oM_w}$$

In this way the aircraft equations of motion reduce to second order Laplace equations which are extremely simple to handle. The time response to any type of input of either elevator or thrust can now be approximated by standard Laplace techniques. The response of airspeed to either thrust or elevator will be controlled by the lightly damped (at this airspeed) phugoid characteristic equation while the angle of attack response to either forcing function will have the characteristic of the short period mode.

At this point a further simplification can be made by examining the short period motion. By substituting the aerodynamic values in the above equations it can be seen that the short period mode has a damping ratio of .35 with a time to one tenth of the initial disturbance value of 5.6 seconds in a period of 6 seconds.

Thus it can be seen that the response to this disturbance will approach an exponentially damped time response rather than an oscillatory type. This leads directly to the well known assumption that the aircraft moment of inertia in the pitch plane can be neglected. In the Laplace equations above it can be reflected by eliminating the S^2 term from the matrix.[1]

The equations then reduce to:

Eqs(2-15)

$$\frac{\alpha}{\eta} = \frac{M_\eta - \frac{Z_\eta M_q}{U_o}}{S(-U_o M_{\dot{w}} - M_q) + Z_w M_q - U_o M_w}$$

$$\frac{\alpha}{T} = \frac{M_T}{S(-U_o M_{\dot{w}} - M_q) + Z_w M_q - U_o M_w}$$

Which are of the form

Eqs(2-16)

$$\frac{\alpha}{\eta} = \frac{C_1}{S + A} \qquad \frac{\alpha}{T} = \frac{C_2}{S + A}$$

[1]This is more evident from the NACA nondimensional form of the equations. The coefficient of the S^2 term is then much smaller than the other terms.

where A is

$$A = \frac{Z_w M_q - U_o M_w}{- U_o M_{\dot{w}} - M_q}$$

For a step input of elevator or thrust, Eqs(2-16) transform to the time domain as

Eqs(2-17) $$\frac{\alpha(t)}{\eta(t)} = \frac{C_1}{A}\left(1 - e^{-At}\right) \qquad \frac{\alpha(t)}{T(t)} = \frac{C_2}{A}\left(1 - e^{-At}\right)$$

Since A is usually large (3.5 in this case) the exponentional term can be neglected. This indicates that the angle of attack response to either thrust or elevator will be approximately constant. In aerodynamic terms,

Eqs(2-18) $$\frac{\alpha(t)}{\eta(t)} = \frac{M_\eta - Z_\eta M_q/U_o}{Z_w M_q - U_o M_w} \qquad \frac{\alpha(t)}{T(t)} = \frac{M_T}{Z_w M_q - U_o M_w}$$

Substituting values gives,

Eqs(2-19) $$\frac{\alpha(t)}{\eta(t)} = -1.74 \qquad \frac{\alpha(t)}{T(t)} = -1.2 \times 10^{-6} \text{rad/lb}$$

The value obtained for $\alpha(t)/\eta(t)$ is within 10% of the value determined previously by analogue results. [9] The low value for $\alpha(t)/T(t)$ shows that the angle of attack response to thrust is negligible. The Power Compensator then can be used only to control the airspeed perturbations and elevator must be used to control the angle of attack perturbations. This is not to say that the airspeed variations will not affect the angle of attack. Any airspeed perturbations will cause the aircraft to seek a new equilibrium angle of attack. The angle of attack loop is necessary to offset the large rise in drag for positive excursions of angle of attack.

The assumption of two independent motions is now carried into the control system of Figure 1 by considering each loop separately. The equations are further simplified by assuming all delay terms are equal to unity. The two simplified loops with their closed loop transfer functions are shown in Figure 4. These functions with aerodynamic terms substituted are shown below.

Eqs(2-20)
$$\frac{u}{u_c} = \frac{-X_T K_u (S + k_u)}{S^2 + (K_u X_T - X_u) S - \frac{g}{U_o} Z_u + X_T K_u k_u}$$

Eqs(2-21)
$$\frac{\alpha}{\alpha_c} = \frac{M_T}{S^2 + (Z_w + M_q) S + Z_w M_q - U_o M_w - M_T K_\alpha}$$

Roots Locus plots for these functions can be easily estimated since they are only second order. Root Locus plots for these functions are shown in Figures 5 and 6 for the aerodynamic values listed in Table III.

The gains can then be chosen from these Root Locus Plots. As a first approximation the gains can be chosen such that the phugoid roots are on the imaginary axis in close proximity to the short period roots. This will give a response of moderate overshoot with the damping of the short period roots. Note that the short period roots are changed only slightly from their basic aircraft position and that the gain of the angle of attack loop affects these roots only to a small degree.

The angle of attack gain can be approximated by considering its desired function of equalizing the drag rise due to the angle of attack. This gives

$$\frac{\partial T_c}{\partial \alpha} = \frac{\partial D}{\partial \alpha} \text{ where } T_c = K_\alpha \alpha_e$$

Therefore $K_\alpha = \frac{1}{2}\rho V^2 S C_{D_\alpha}$. This gives an approximation of 24,000 lb/rad. The values of the other gains chosen were:

$$K_u = 300 - 500 \text{ lb/ft/sec}$$

$$k_u = .05\text{/sec}$$

The effect of the gains on the overall motion can be analized from the simplified Root Locus. K_u will mainly affect the time constant of the response. Thus increasing K_u will increase the speed of the response with a resulting increase in overshoot. The effect of K_α is small but will increase the overshoot as it is increased. The effect of k_u is difficult to analize without a detailed study of the Root Locus. In general, increasing k_u will increase the overshoot and can cause a second oscillatory mode to appear at about the same frequency as the short period mode. This would give an undesirable response.

The gains chosen as first approximations are:

$$K_u = 400 \text{ lb/ft/sec}$$

$$k_u = .05\text{/sec}$$

$$K_\alpha = 20,000 \text{ lb/rad}$$

$$K_u k_u = 2.0 \text{ lb/ft}$$

2.8 Co-ordinated Elevator Input

In order to stabalize the aircraft motion at a steady value of airspeed, elevator must be used to control the flight path. This input will simulate the action of either an autopilot feedback or the action of the pilot. The gain for this elevator/airspeed loop is obtained from the following analysis;

$$V^2 = (U_o + u)^2 = \frac{W/S}{\frac{1}{2}\rho C_L} \triangleq U_o^2 + 2uU_o$$

then

$$2U\,du = \frac{-W/S}{\frac{1}{2}\rho C_L^2}\, dC_L$$

or

$$2uU = \frac{-W/S}{\frac{1}{2}\rho C_L^2}\, C_{L\alpha}\alpha$$

therefore

$$u = \frac{-W/SC_{L\alpha}}{\rho U C_L{}^2}\alpha$$

For the given flight condition

$$u(\text{kts}) = -140.0\,\alpha(\text{rad}) = -6.32\,\alpha(\text{deg})$$

From Eqs 19, $\alpha = -1.74\,\eta$

Then $u(\text{kts}) = 11.0\,\eta$ (deg)

3. Analogue Simulation.

In order to check the validity of the foregoing analysis, the full equations (Eqs. 2-4) were solved on an analogue computer and the gains of the Power Compensator "optimized" for horizontal and vertical gust inputs. A Donner 3100 computer was used in the analysis. The analogue wiring diagrams are shown in Figures 7 and 8. All analogue program information is given in the Appendix. The values given for the flight condition in Table III were used in the program. Figures 9, 10, and 11 show the response of the basic airframe to horizontal and vertical gusts and to a step elevator input. The motion shows the phugoid response with a period of 28 seconds which compares with the calculated value of 30 seconds. The short period motion is difficult to see because of the heavy damping but the period is less than 10 seconds. Figure 11 clearly shows the relation between elevator input and angle of attack. The angle of attack rapidly takes on a small phugoid oscillation about a constant value of - 1.8 deg for an elevator input of 1 deg. This compares well with Eq(2-19).

3.1 Basic Airframe

Figures 12, 13, and 14 show the response of the basic airframe to sinusoidal horizontal gusts with periods close to those of the short period and phugoid motion. Figures 12 and 13 show that the response to the short period gust is of small magnitude and composed of the short period motion superimposed on the longer period phugoid. As the period of the phugoid mode is reached, Figure 14 shows that the response becomes divergent. The same is true for vertical gust inputs shown in Figures 15 and 16.

3.2 Basic Airframe with Power Compensator

In order to check the validity of the gain analysis given in 2.7, the response of the airspeed motion for various values of loop gains is given in Figures 17 through 20. Figure 17 shows that K_u affects the damping of the motion and is the main controlling gain. Figure 18 shows the decrease of damping with increasing integral gain, $K_u k_u$. Figures 19 and 20 show the small effect of the angle of attack gain, K_α, on both horizontal and vertical gusts. Since the power compensator will have the most effect on the air-speed loop, the airspeed response was used to choose the "best" gains.

Figures 21 through 25 show the response of all the important parameters to various gains for the most unfavorable gust condition; - 5 kt horizontal and + 5 kt vertical gust. Important aspects of these Figures are summarized below.

TABLE V
ANALOGUE DATA

Fig	Overshoot	Time to within 1 kt	Maximum T required	γ after 10 sec
21	65%	6.6 sec	2000 lbs	- 1.5 deg
22	80	8.5	1800	- 1.3
23	70	4.8	2800	- 1.5
24	85	9.5	3500	- 1.5
25	80	9.0	1400	- 1.2

The gains from Figure 21 were considered as the most favorable. The overshoot is somewhat misleading since it will be reduced considerably by the pilots application of elevator to correct the flight errors. However, this overshoot could be reduced by series compensation.

The gains of Figure 21 were used for all the remaining runs containing the power compensator and are listed below.

$$K_u = 400 \text{ lb/ft/sec}$$

$$K_u k_u = 2.0 \text{ lb/ft}$$

$$K_\alpha = 10,000 \text{ lb/rad}$$

Figures 26 through 30 show the response of the airframe and power compensator to sinusoidal gust inputs. They show that the compensator eliminates the phugoid divergence but Figure 29 indicates that the system tends to magnify the response for vertical gusts with a period of 9.6 seconds. However this period should be long enough to be controlled by the pilot and the airspeed perturbation is not serious. Figures 31 and 32 show the response to an airspeed command with coordinated elevator input as discussed in 2.8. These Figures show that the overshoot is reduced to 26% and the glide slope variation to within .5 degrees.

4. Root Locus Analysis.

In order to compare the simplified Root Locus plots with those for the complete system, the characteristic equation for the complete system was evaluated from Eq(2-8). This is shown in the Appendix. Root Locus plots of this polynomial were then developed for several values of loop gains. These plots are shown in Figures 33 and 34.

5. Conclusions.

1. The airspeed/angle of attack power compensator would be a feasible system to employ especially where the use of a full auto-pilot is anticipated.

2. The simplified method of setting loop gains by considering the control loops as separate functions of the phugoid and short period motions will give valid results.

3. The response of the system can be approximated by considering the separate root locus plots of each loop.

4. The system may have to be improved because of the excessive

overshoot response of airspeed. This could be done by series compensation.

5. The effect of the gains on the Root Locus of the complete system compares qualitatively to the effect of the gains on the two loops considered separately.

6. The power compensator will have negligible effect on the short period motion and can be thought of as an airspeed (phugoid) damping device. This will be true whenever the thrust axis is close to the body axis giving small values for M_T and Z_T.

6. Recommendations for Future Study.

1. The effects of the non-linear aspects of $C_{D\alpha}$ on the pitch motion feedback gain requirements for all three power compensators studied thus far should be the next field of study.

2. An investigation to determine which of the three systems studied thus far would give the best results when combined with an elevator autopilot would indicate an optimum combination for a complete system.

3. A complete analysis of a flight control system should include the effects of turbulent air gusts and aeroelasticity. To do this type of an analysis by analogue simulation would require an excessive number of components. Since the problem of response to random frequencies is best analyzed by frequency response methods, a digital computer program would give the most convenient approach. This type of a problem could also be analyzed on a "hybrid computer system" by using the analogue to solve the equations and the digital to introduce the non-linearities.

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6. Anon., Model F8U-2N Airplane Landing Configuration Data, Chance Vought Corporation, Division of Ling-Temco-Vought, Inc., Ltr Ser 2-5330/212048 with enclosures, Sept., 1962.

7. Anon., Comparison of Two Automatic Throttle Controllers for the F8U-2NE Aircraft, U. S. Naval Air Development Center, EDL Report No NADC-ED-L6292, 1962.

8. Bell, Gerald R., An Investigation of the Effect of Auto-Throttle Devices on Aircraft Control in the Carrier Landing Approach, U. S. Naval Postgraduate School, B360, 1963.

9. Evans, Ronald E. and Schuppe, Robert H., An Investigation of the Effect of Simplified Equations of Motion and Various Inputs on Automatic-Throttle Devices, U. S. Naval Postgraduate School, E775, 1964.

10. Toll, Thomas A. and Queijo, M. J., Approximate Relations and Charts for Low Speed Stability Derivatives of Swept Wings, Technical Report No 1581, 1948.

TABLE I

DEFINITIONS OF DIMENSIONAL AERODYNAMIC DERIVATIVES

X_u	$= \frac{1}{m}\frac{\partial X}{\partial u}$	$\frac{1}{sec}$
X_w	$= \frac{1}{m}\frac{\partial X}{\partial w}$	$\frac{1}{sec}$
X_q	$= \frac{1}{m}\frac{\partial X}{\partial q}$	$\frac{ft}{sec\text{-}rad}$
X_T	$= \frac{1}{m}\frac{\partial X}{\partial T}$	$\frac{ft}{sec^2\text{-}lb}$
X_η	$= \frac{1}{m}\frac{\partial X}{\partial \eta}$	$\frac{ft}{sec^2\text{-}rad}$
Z_u	$= \frac{1}{m}\frac{\partial Z}{\partial u}$	$\frac{1}{sec}$
Z_w	$= \frac{1}{m}\frac{\partial Z}{\partial w}$	$\frac{1}{sec}$
Z_q	$= \frac{1}{m}\frac{\partial Z}{\partial q}$	$\frac{1}{sec\text{-}rad}$
Z_T	$= \frac{1}{m}\frac{\partial Z}{\partial T}$	$\frac{ft}{sec^2\text{-}lb}$
Z_η	$= \frac{1}{m}\frac{\partial Z}{\partial \eta}$	$\frac{ft}{sec^2\text{-}rad}$
M_u	$= \frac{1}{B}\frac{\partial M}{\partial u}$	$\frac{1}{sec\text{-}ft}$
M_w	$= \frac{1}{B}\frac{\partial M}{\partial w}$	$\frac{1}{sec\text{-}ft}$
$M_{\dot{w}}$	$= \frac{1}{B}\frac{\partial M}{\partial \dot{w}}$	$\frac{1}{ft}$
M_q	$= \frac{1}{B}\frac{\partial M}{\partial q}$	$\frac{1}{sec\text{-}rad}$
M_T	$= \frac{1}{B}\frac{\partial M}{\partial T}$	$\frac{1}{sec^2\text{-}lb}$
M_η	$= \frac{1}{B}\frac{\partial M}{\partial \eta}$	$\frac{1}{sec^2\text{-}rad}$

TABLE II

CONVERSION OF DERIVATIVES

$$X_u = \frac{\rho S V}{m} (-C_D - C_{Du})$$

$$X_w = \frac{\rho S V}{2m} (C_L - C_{D\alpha})$$

$$X_\eta = \frac{\rho S V^2}{2m} C_{D\eta}$$

$$X_T = \frac{\cos \nu}{m}$$

$$Z_u = \frac{\rho S V}{m} (-C_L)$$

$$Z_w = \frac{\rho S V}{2m} (-C_{L\alpha} - C_D)$$

$$Z_\eta = \frac{\rho S V^2}{2m} (-C_{L\eta})$$

$$Z_T = \frac{\sin \nu}{m}$$

$$M_u = \frac{\rho S V}{B} (-C_T z_T)$$?

$$M_w = \frac{\rho S V c}{2B} (C_{m\alpha})$$

$$M_{\dot{w}} = \frac{\rho S c^2}{4B} (C_{m\dot{\alpha}})$$

$$M_q = \frac{\rho S V c^2}{4B} (C_{mq})$$

$$M_\eta = \frac{\rho S V^2 c}{2B} (C_{m\eta})$$

$$M_T = \frac{-z_T}{B}$$

TABLE III

AIRCRAFT PARAMETERS

S_W	375 ft^2
z_T	-.437 ft
l_t	14.08 ft
c	11.78 ft
B	96,000 slug-ft^2

FLIGHT CONDITIONS

V	$1.15V_s$	234 ft/sec
cg	24% MAC	
W	22,000 lbs	
θ_o	8.1 deg	

AERODYNAMIC DERIVATIVES

X_u	-.060 1/sec
X_w	-.01419 1/sec
X_q	0
X_T	.00145 ft/lb-sec^2
X_η	-1.64 ft/sec^2-rad
Z_u	-.2655 1/sec
Z_w	-.4265 1/sec
Z_q	0
Z_T	-2.17×10^{-5} ft/sec^2-lb
Z_η	-19.1 ft/sec^2-rad
M_u	.000185 1/sec-ft
M_w	-.004858 1/sec-ft
$M_{\dot{w}}$	-1.772×10^{-4} 1/ft
M_q	-.3384 1/sec-rad

TABLE III (CONT)

M_T	-4.55×10^{-6} 1/lb-sec
M_η	-2.25 1/rad-sec^2

-.000217 ft/sec^{-2} lb

TABLE IV

TRANSFER FUNCTIONS

Airframe

Characteristic equation (CH)

$$s^4 + .8659s^3 + 1.3.3s^2 + .0608s + .0423$$

$$\frac{u}{T} = \frac{.00145(s^3 + .8059s^2 + 1.378s + .020)}{CH}$$

$$\frac{u}{\eta} \quad \frac{.198(s^2 + 443.9s + 161.68)}{CH}$$

$$\frac{\alpha}{T} \quad \frac{-6.16\times10^{-6}(s^2 + .0772s + .0267)}{CH}$$

$$\frac{\alpha}{\eta} \quad \frac{-.0596(s^3 + 41.93s^2 + 1.70s + 78.23)}{CH}$$

Engine

$$\frac{T}{T_f} = \frac{1}{1 + \tau_e S} \qquad \tau_e = 1.15 \text{ sec}$$

Angle of Attack Measurement Lag

$$D_\alpha = \frac{1}{1 + \tau_a S} \qquad \tau_a = 0.5 \text{ sec}$$

Airspeed Measurement Lag

$$D_u = \frac{1}{1 + \tau_s S} \qquad \tau_s = 0.1 \text{ sec}$$

Power Compensator

$$\frac{T_c}{\alpha_e} = K_\alpha$$

$$\frac{T_c}{u_e} = K_u\left(1 + \frac{k_u}{S}\right)$$

Power Compensator Servo Lag

$$\frac{T_f}{T_c} = \frac{1}{1 + \tau_f S} \qquad \tau_f = 0.2 \text{ sec}$$

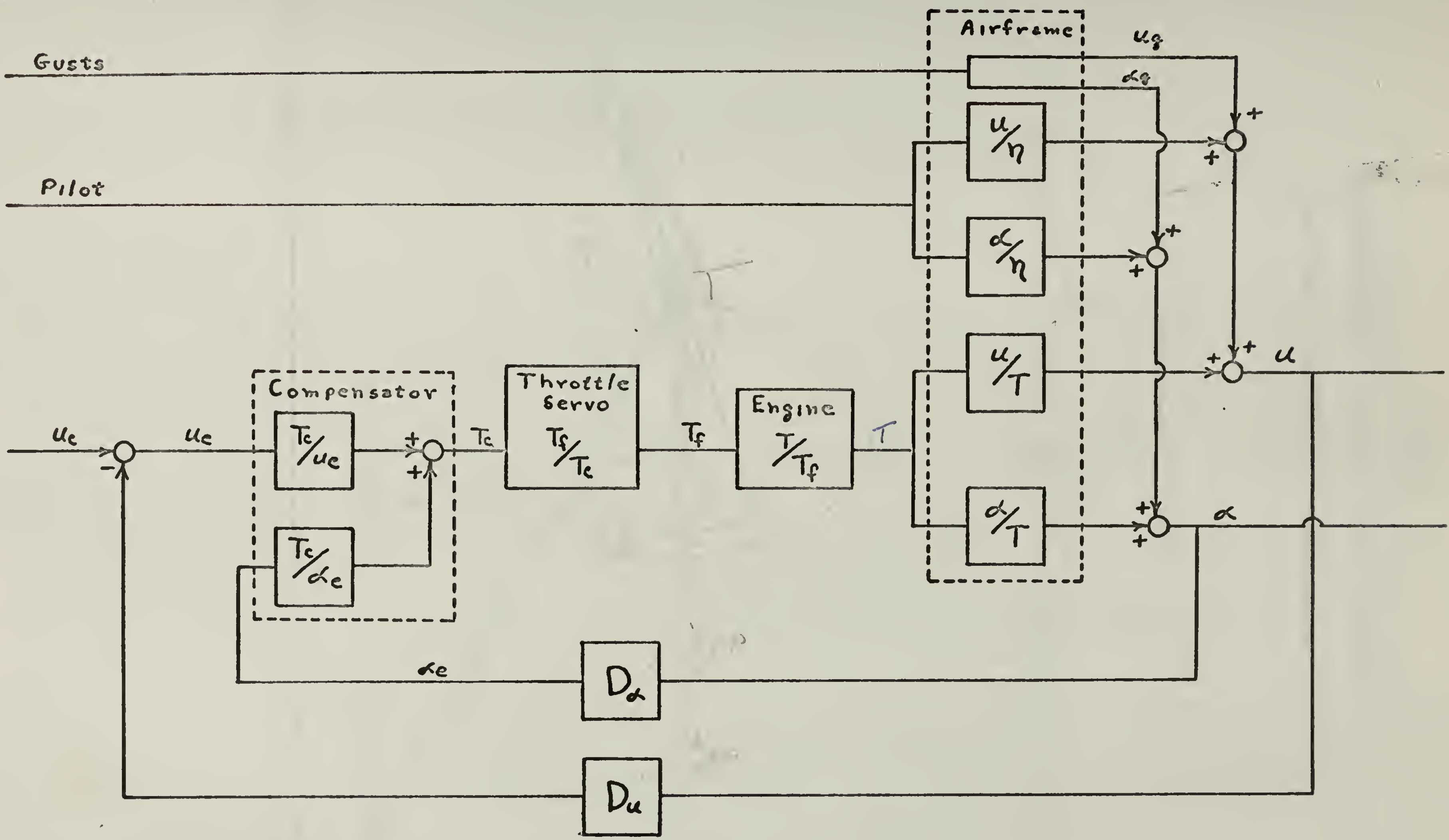


FIGURE 1

BLOCK DIAGRAM OF COMPLETE SYSTEM

FIGURE 2

VELOCITY AND ANGULAR RELATIONSHIPS

Arrows show positive direction

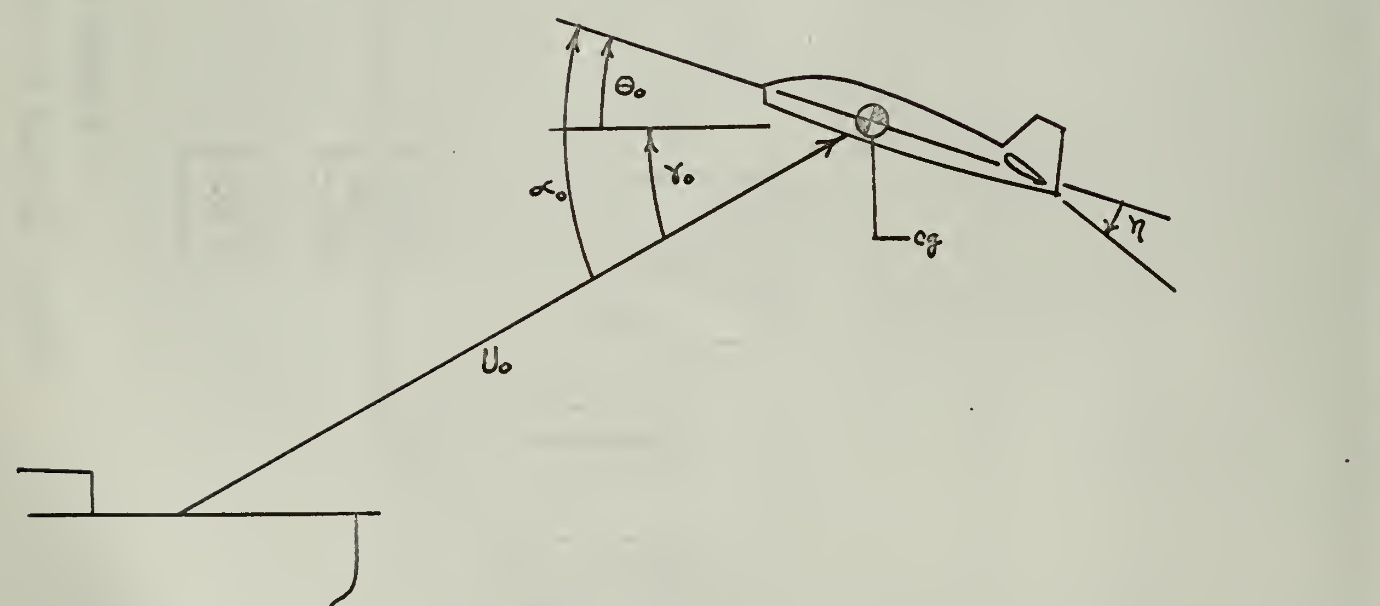
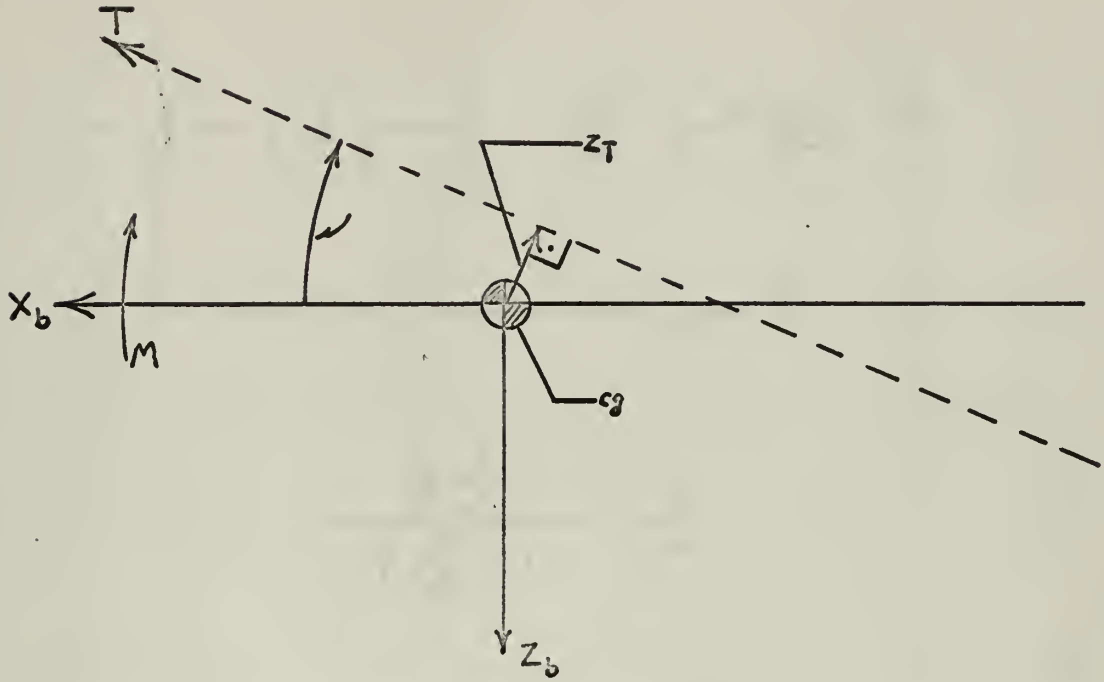


FIGURE 3

DERIVATION OF THRUST DERIVATIVES



Arrows indicate positive direction

$$X_T = \frac{\cos \nu}{m}$$

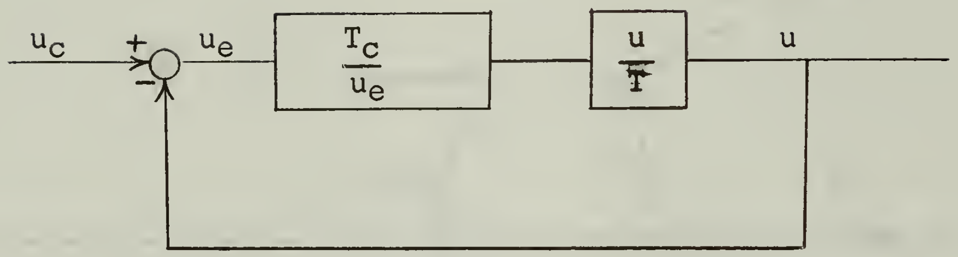
$$Z_T = -\frac{\sin \nu}{m}$$

$$M_T = \frac{-z_T}{B}$$

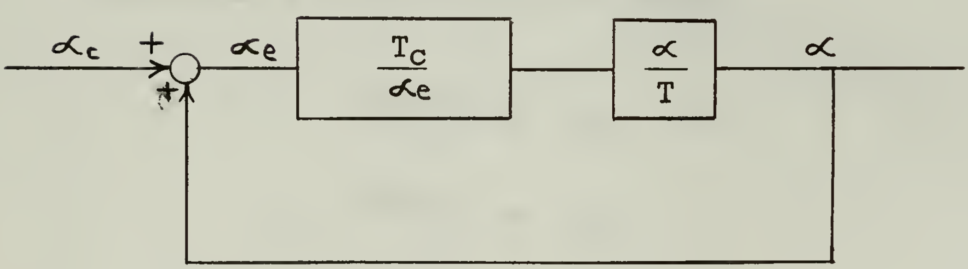
$$\nu = 0.85 \text{ deg}$$

FIGURE 4

SIMPLIFIED LOOPS



$$\frac{u}{u_c} = \frac{\frac{T_c}{u_e}\,\frac{u}{T}}{1 + \frac{T_c}{u_e}\,\frac{u}{T}}$$



$$\frac{\alpha}{\alpha_c} = \frac{\frac{T_c}{\alpha_e}\,\frac{\alpha}{T}}{1 + \frac{T_c}{\alpha_e}\,\frac{\alpha}{T}}$$

FIGURE 5

ROOT LOCUS
Simplified Airspeed Loop - k_u Varies

Basic Airframe Phugoid Roots Δ

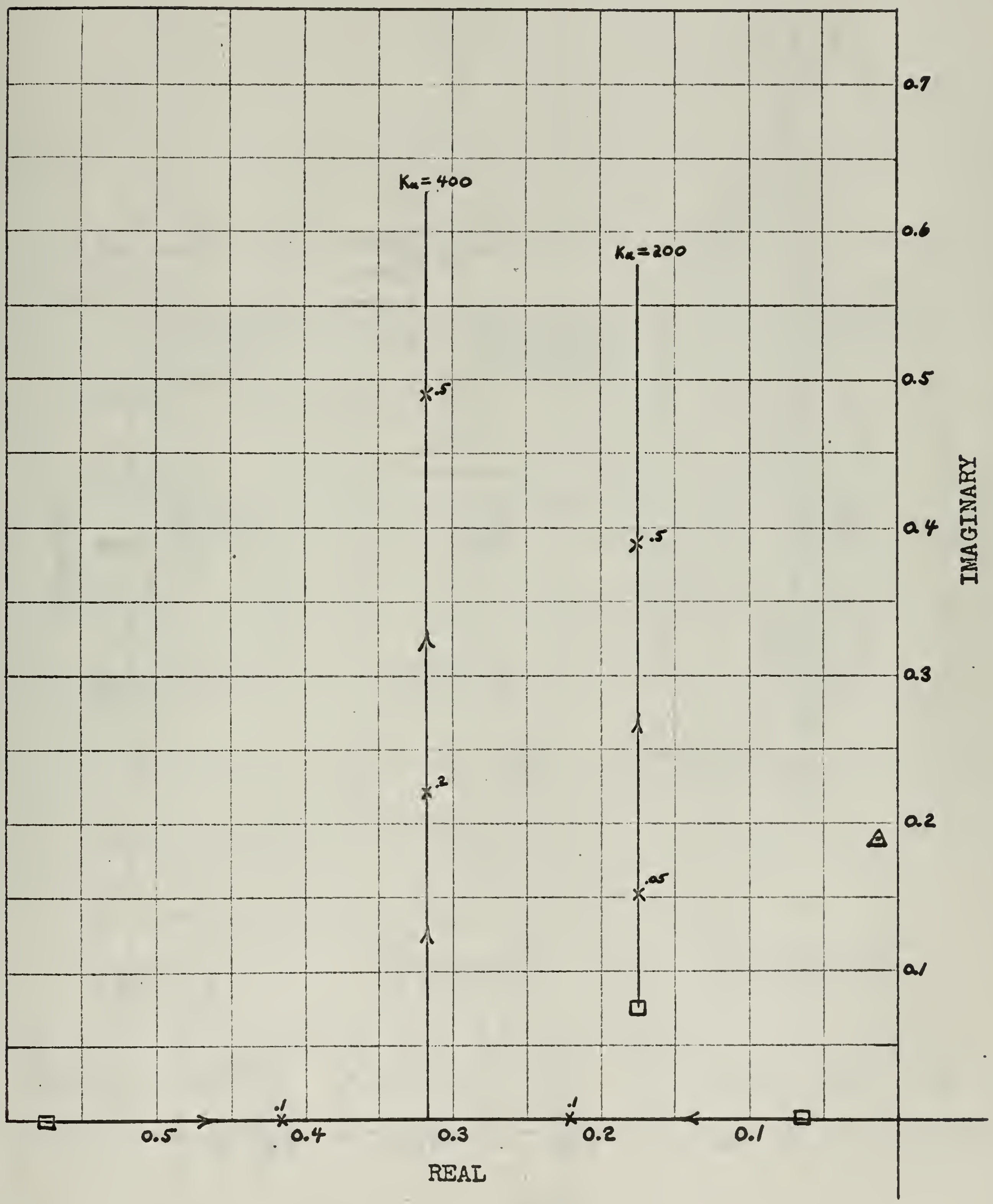
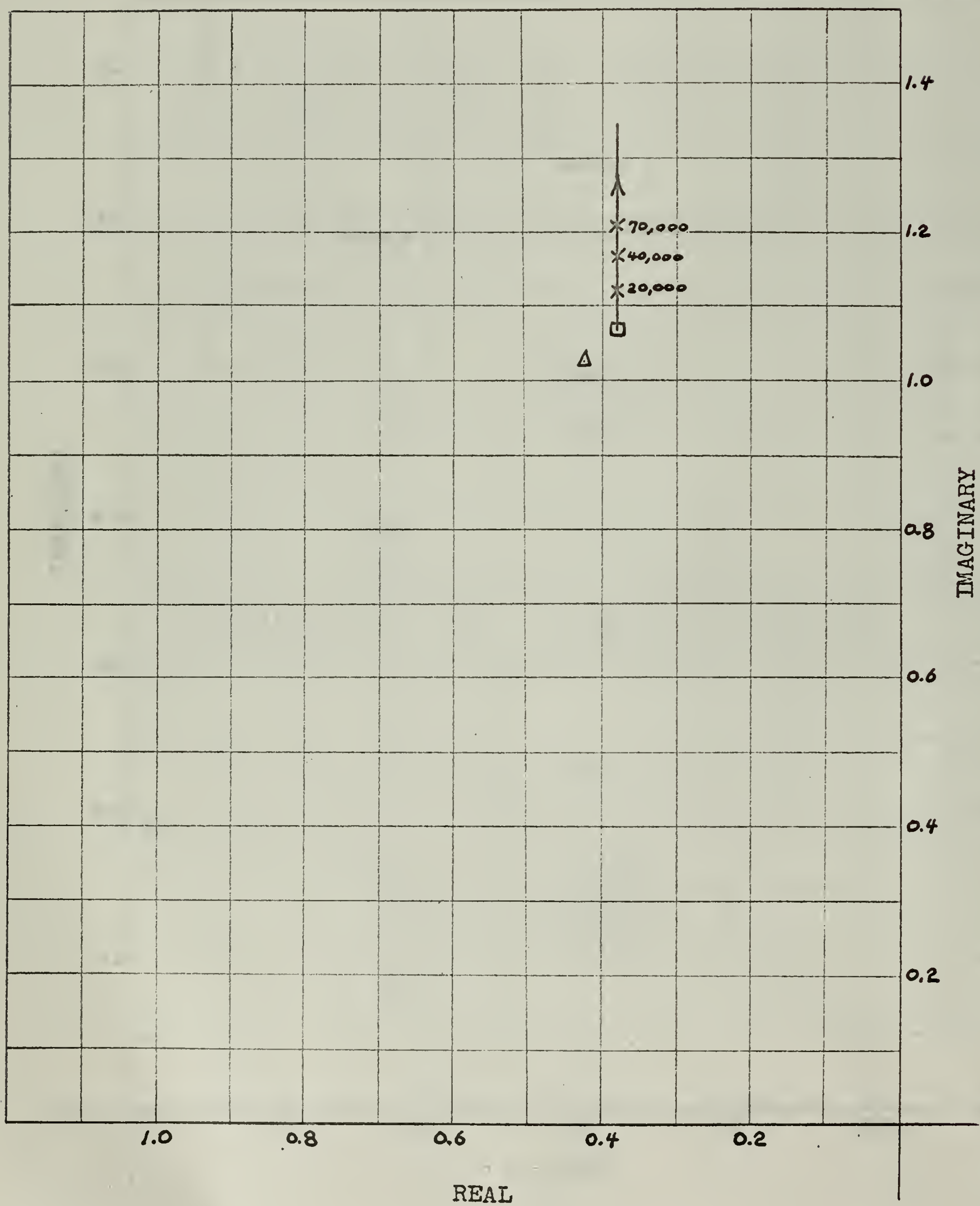


FIGURE 6

ROOT LOCUS
Simplified Angle of Attack Loop - K_α Varies

Basic Airframe Short Period Roots △



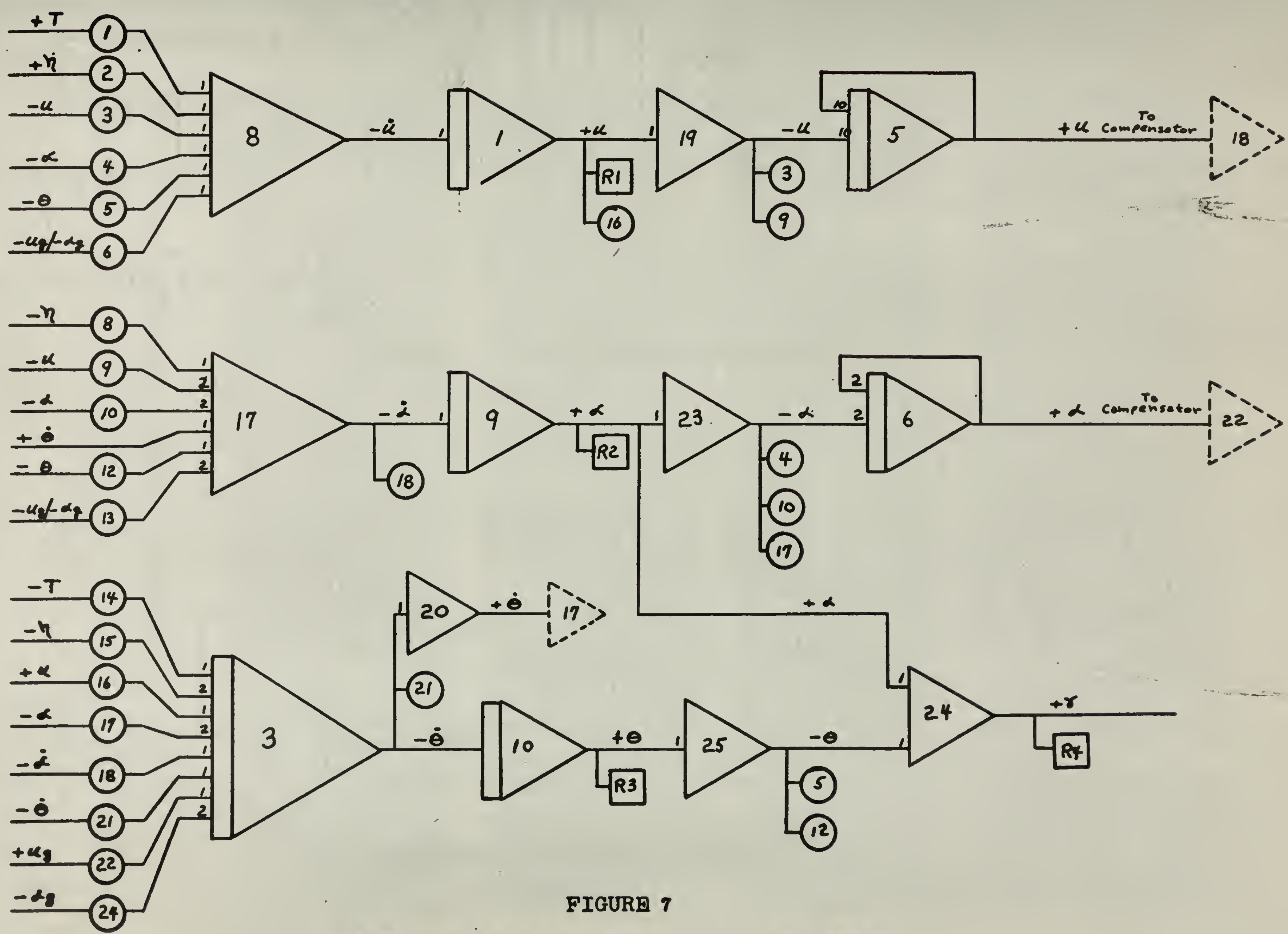


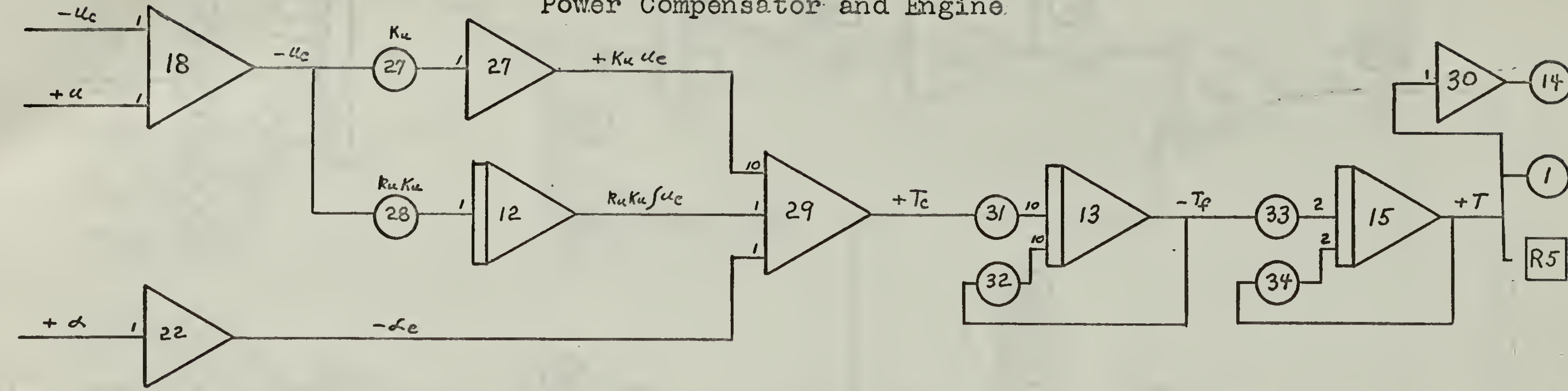
FIGURE 7

ANALOGUE SIMULATION CIRCUITS - Airframe

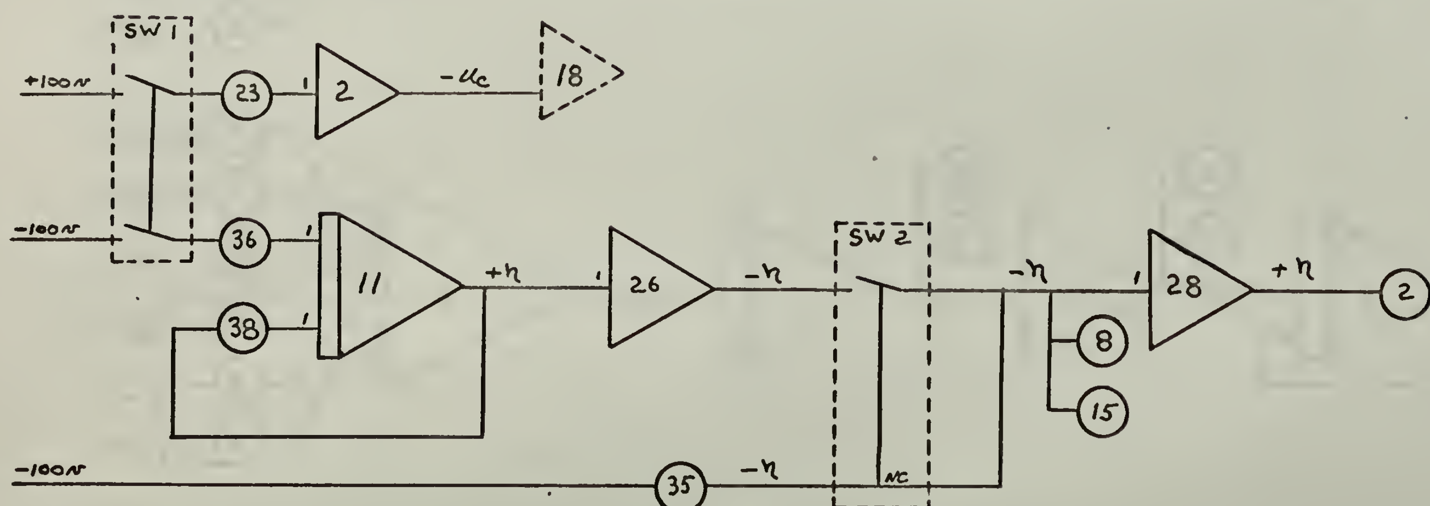
FIGURE 8

ANALOGUE SIMULATION CIRCUITS

Power Compensator and Engine



Airspeed - Elevator Command System



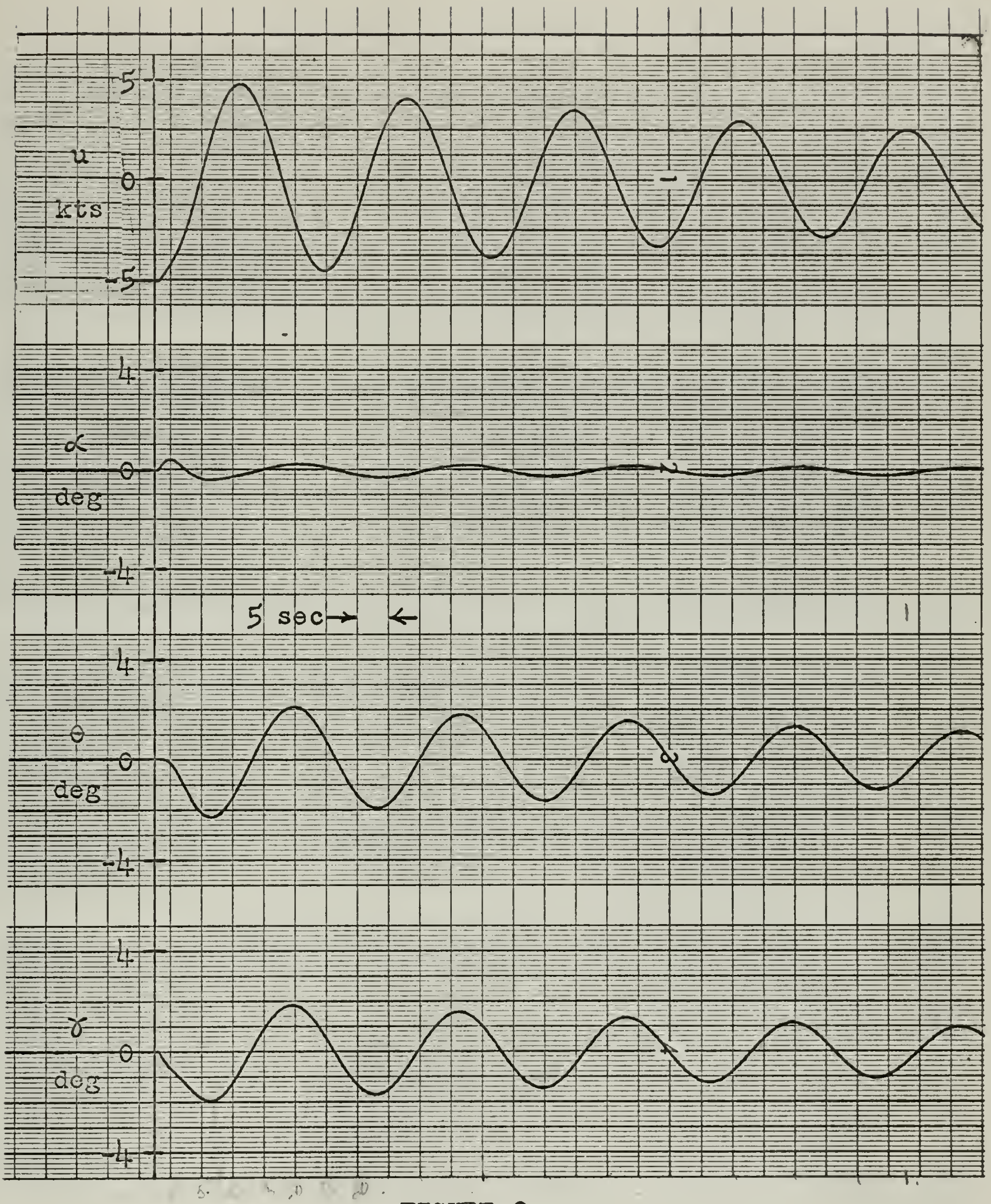


FIGURE 9

Basic Airframe -5 kts Horizontal Gust Input

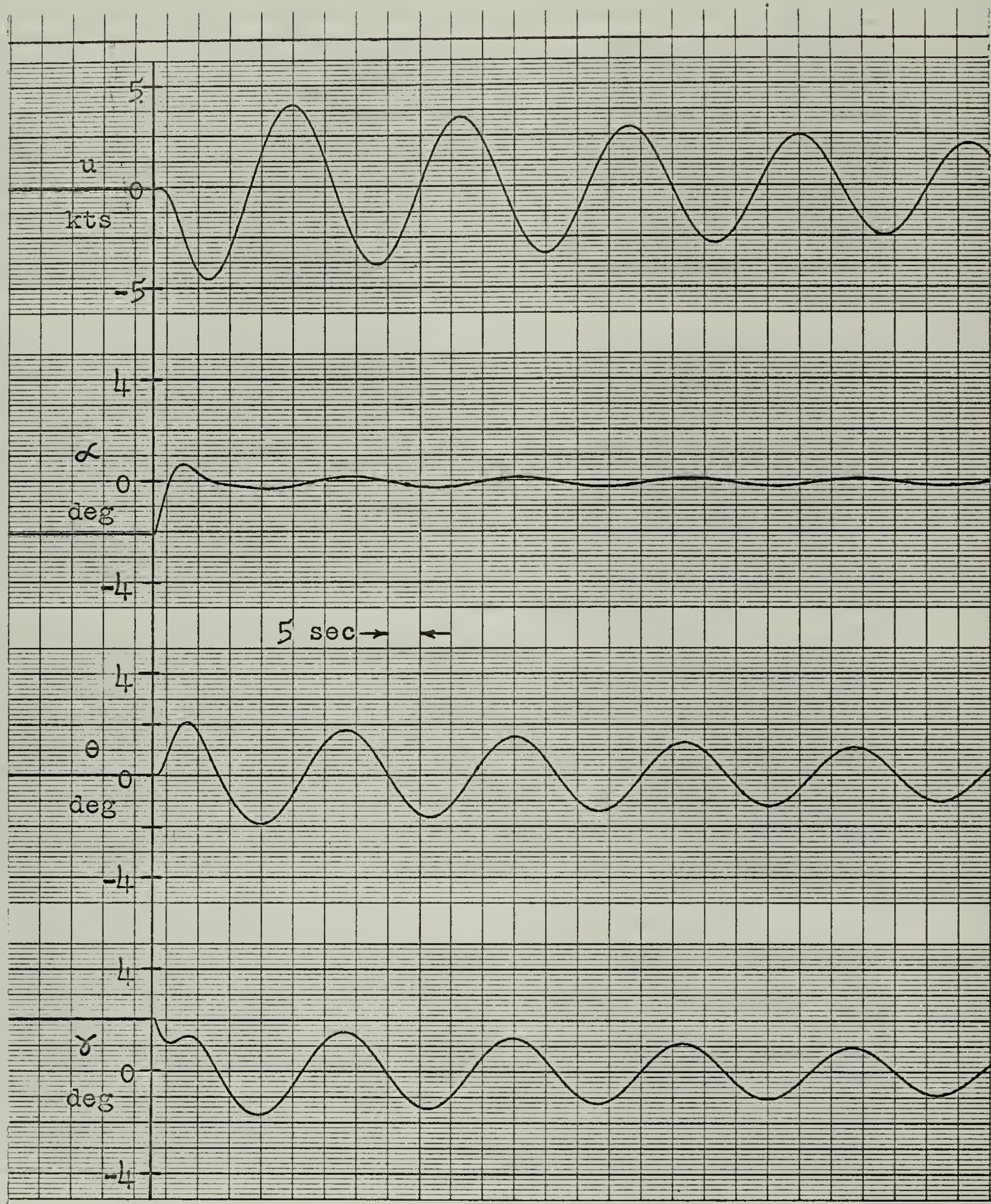


FIGURE 10

Basic Airframe -5 kts(α=2.1°) Vertical Gust Input

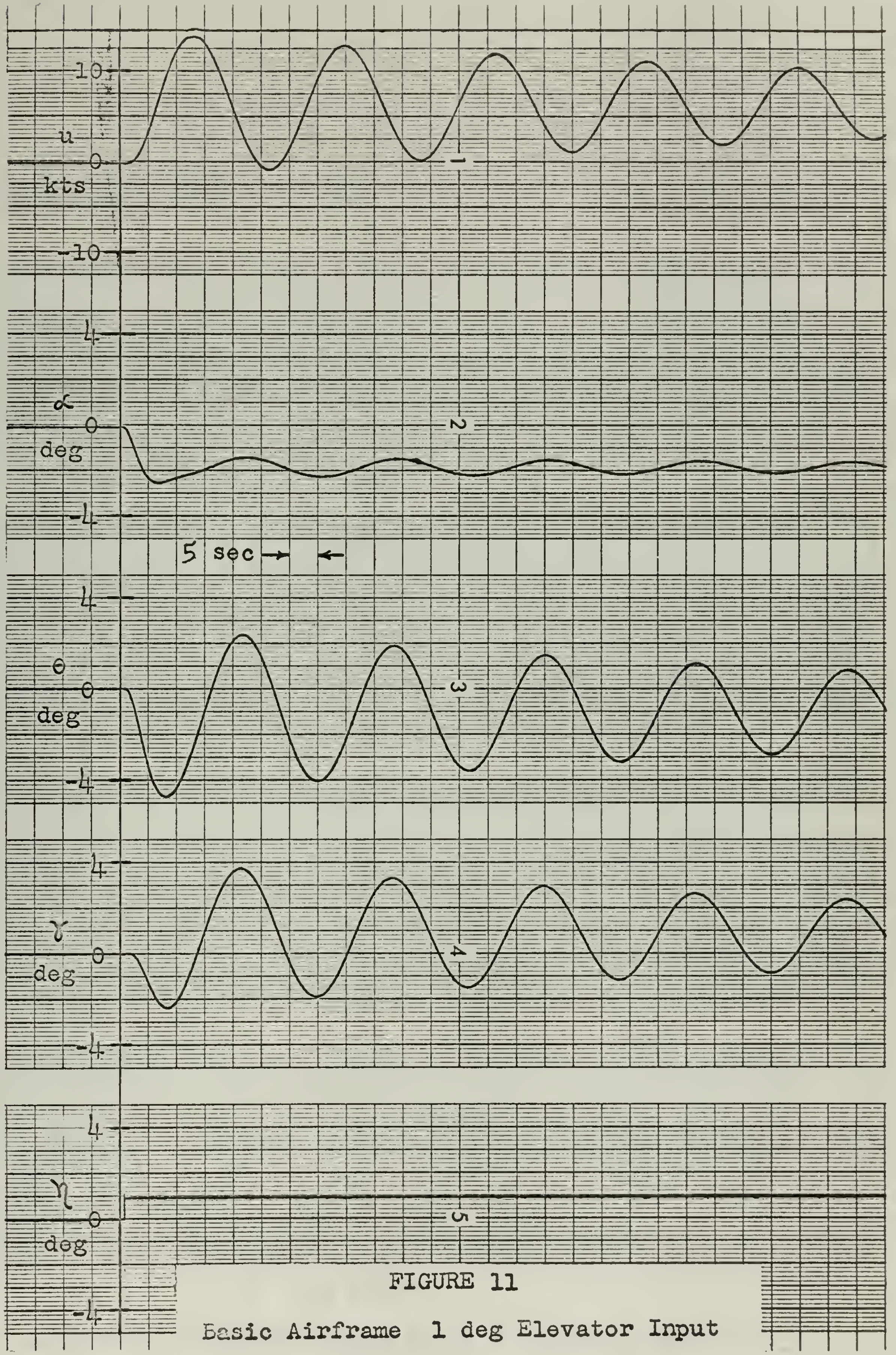


FIGURE 11

Basic Airframe 1 deg Elevator Input

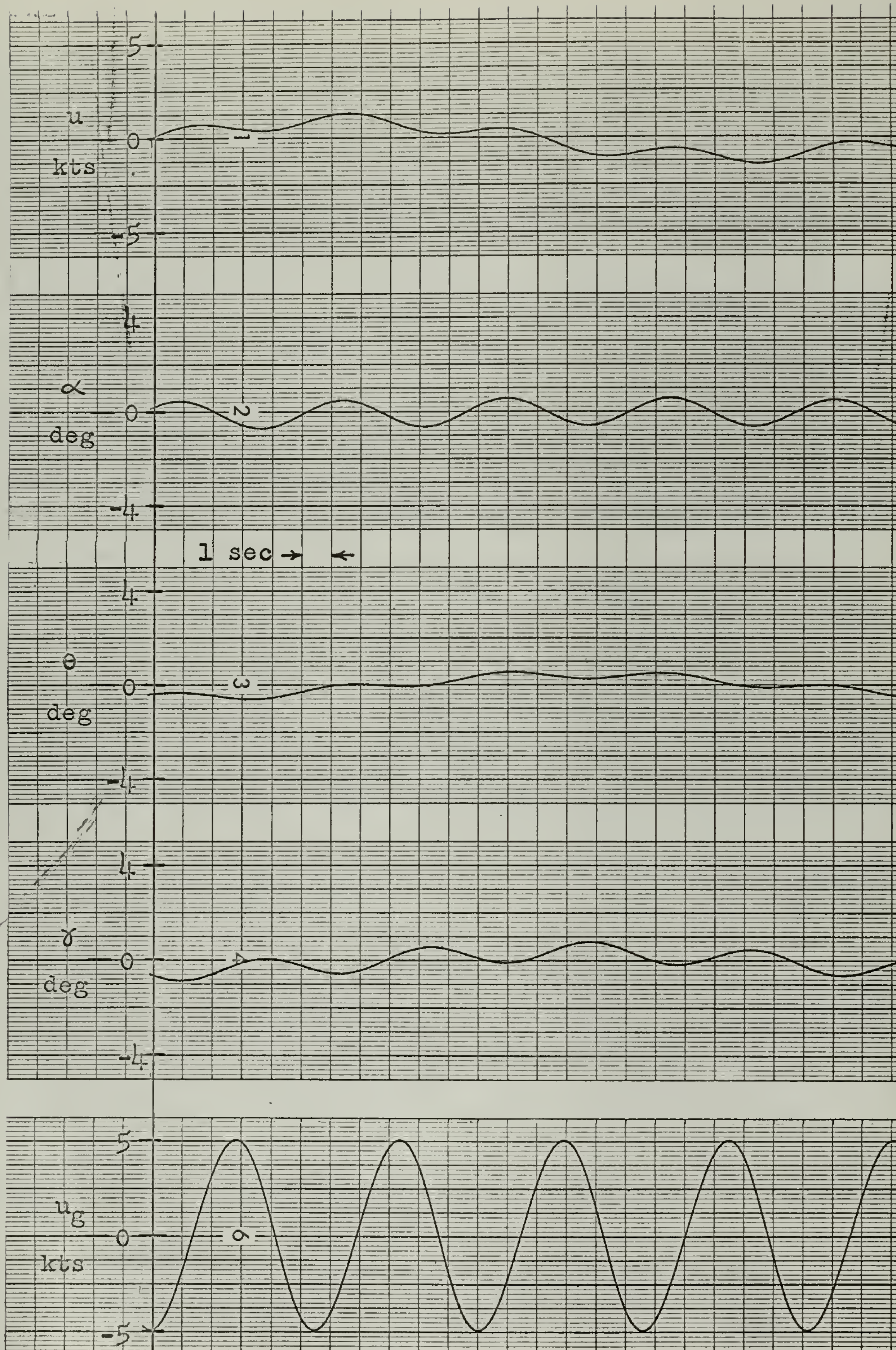


FIGURE 12

Basic Airframe - Sinusoidal Horizontal Gust Input
P = 5.6 sec

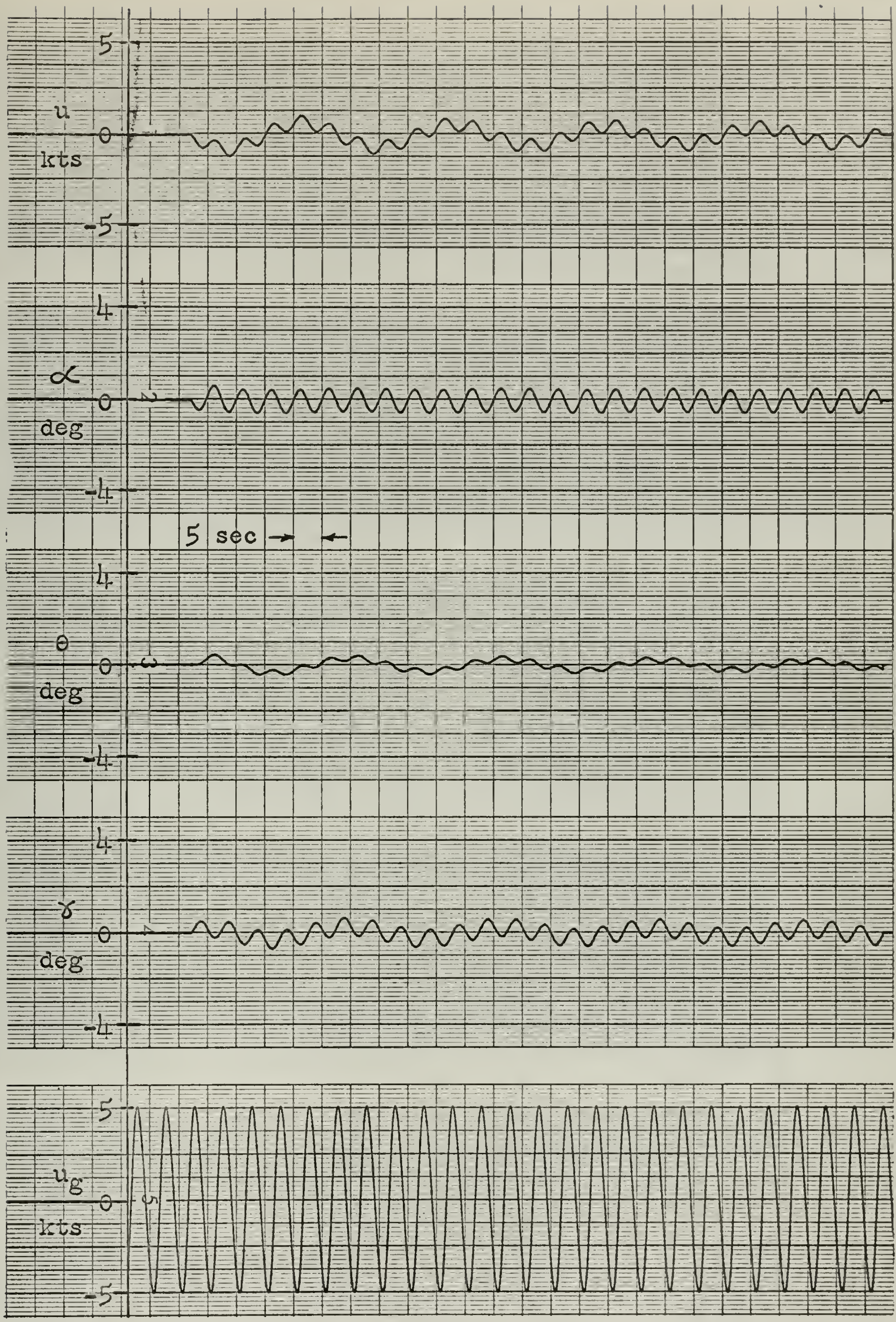


FIGURE 13

Basic Airframe - Sinusoidal Horizontal Gust Input
P = 6.0 sec

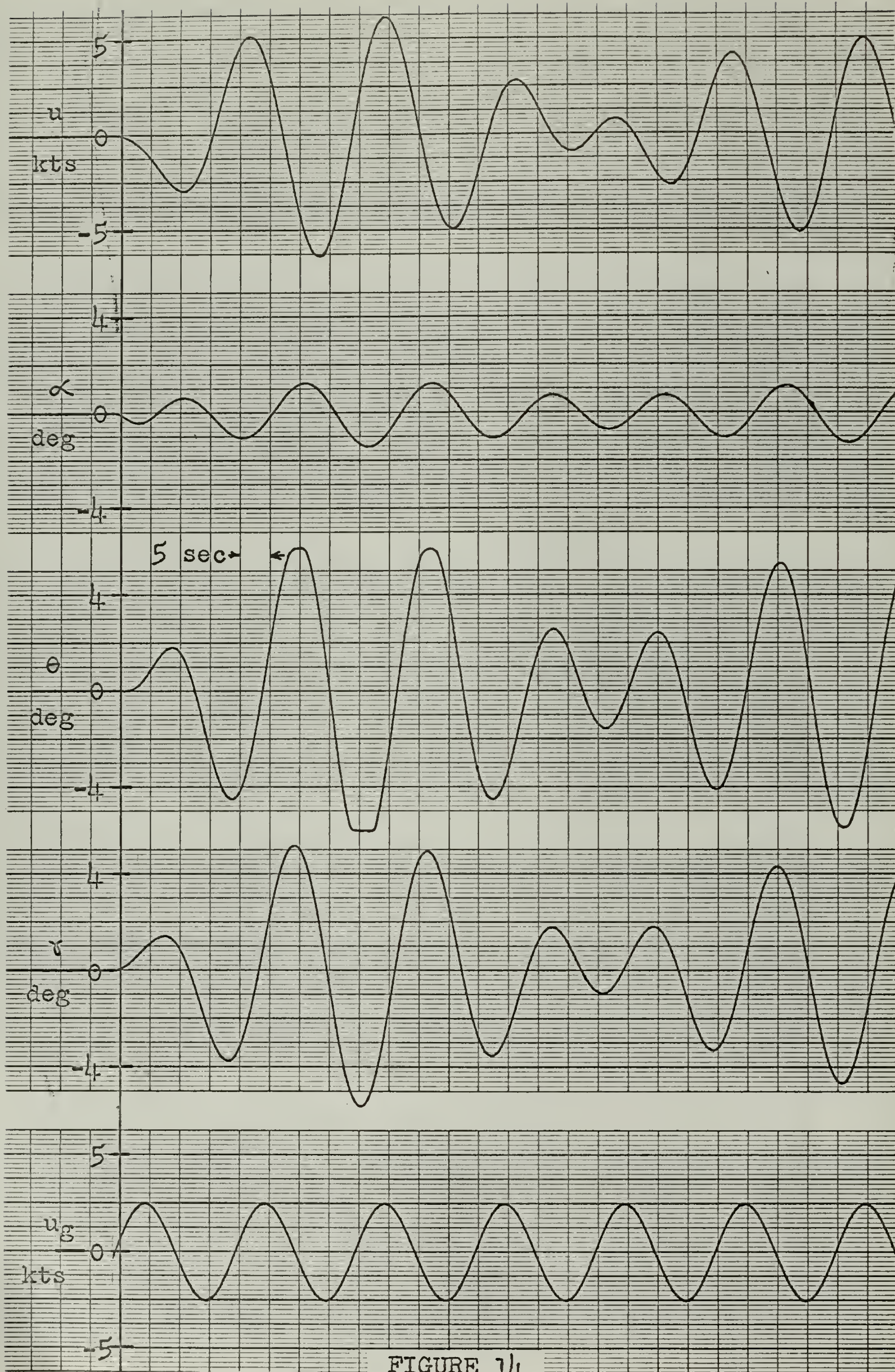


FIGURE 14

Basic Airframe - Sinusoidal Horizontal Gust Input
P = 20 sec

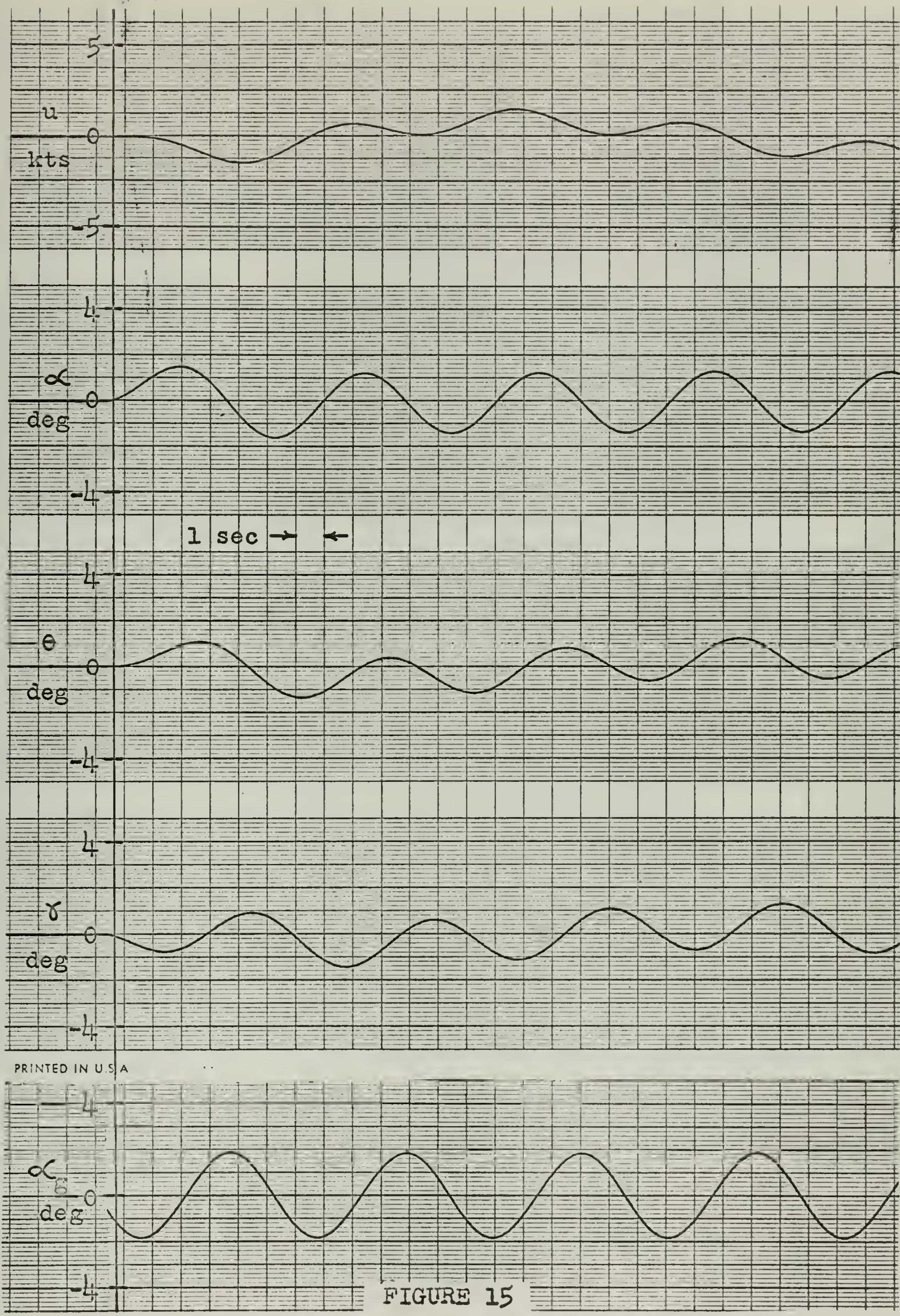


FIGURE 15

Basic Airframe - Sinusoidal Vertical Gust Input
P = 6.0 sec

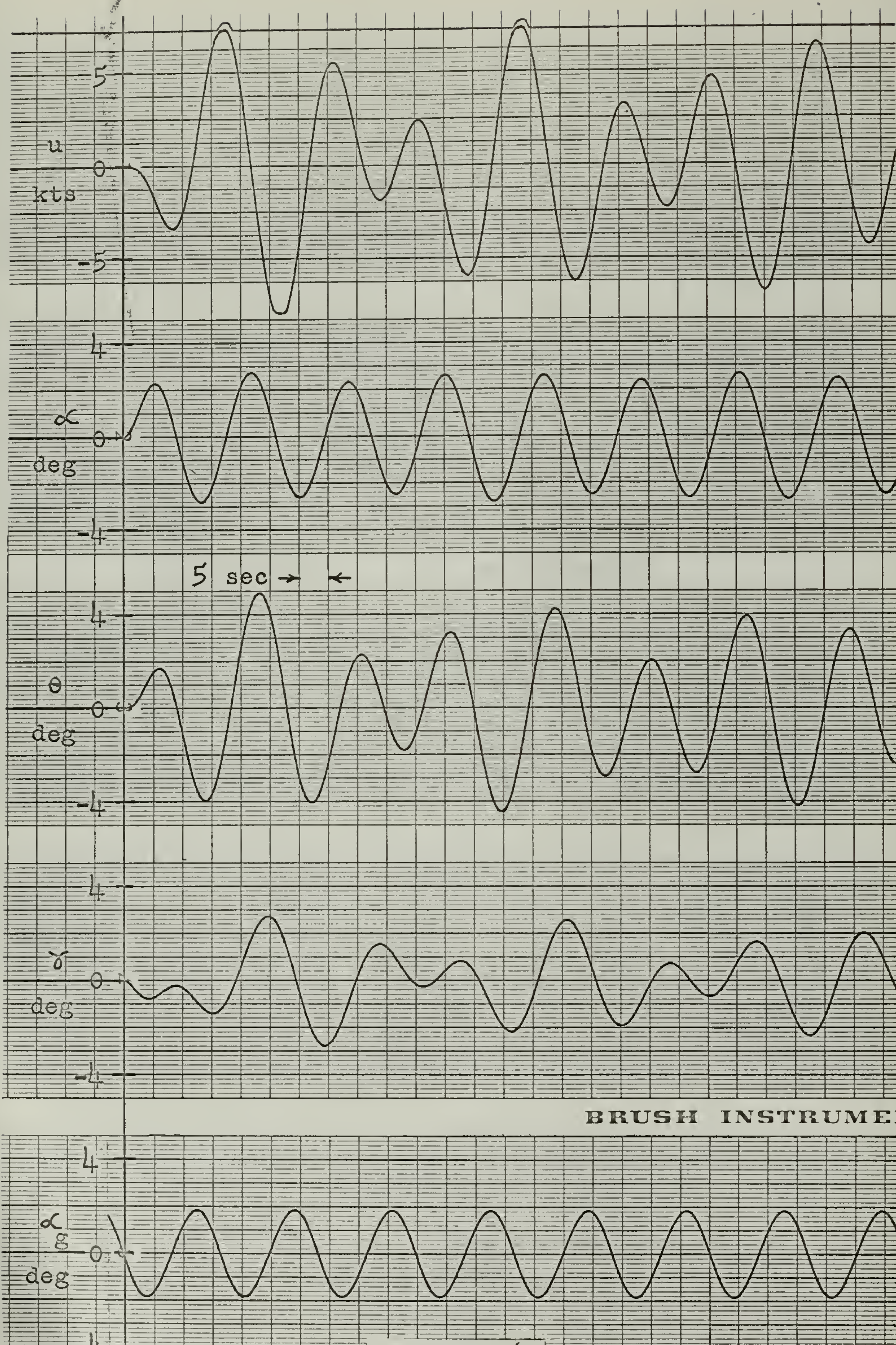


FIGURE 16

Basic Airframe - Sinusoidal Vertical Gust Input
P = 17.5 sec

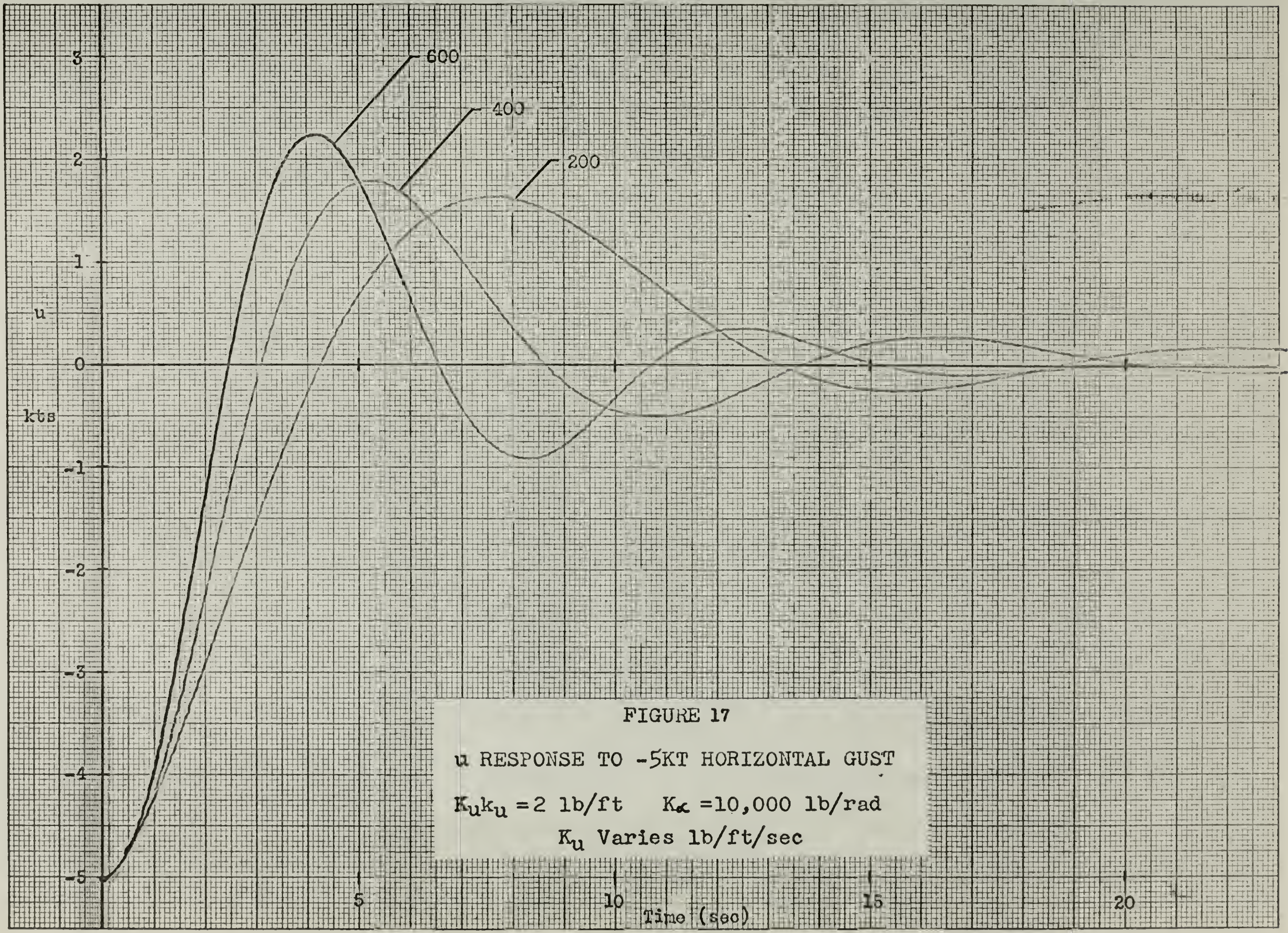


FIGURE 17

u RESPONSE TO -5KT HORIZONTAL GUST

$K_u k_u = 2$ lb/ft $K_\alpha = 10{,}000$ lb/rad

K_u Varies lb/ft/sec

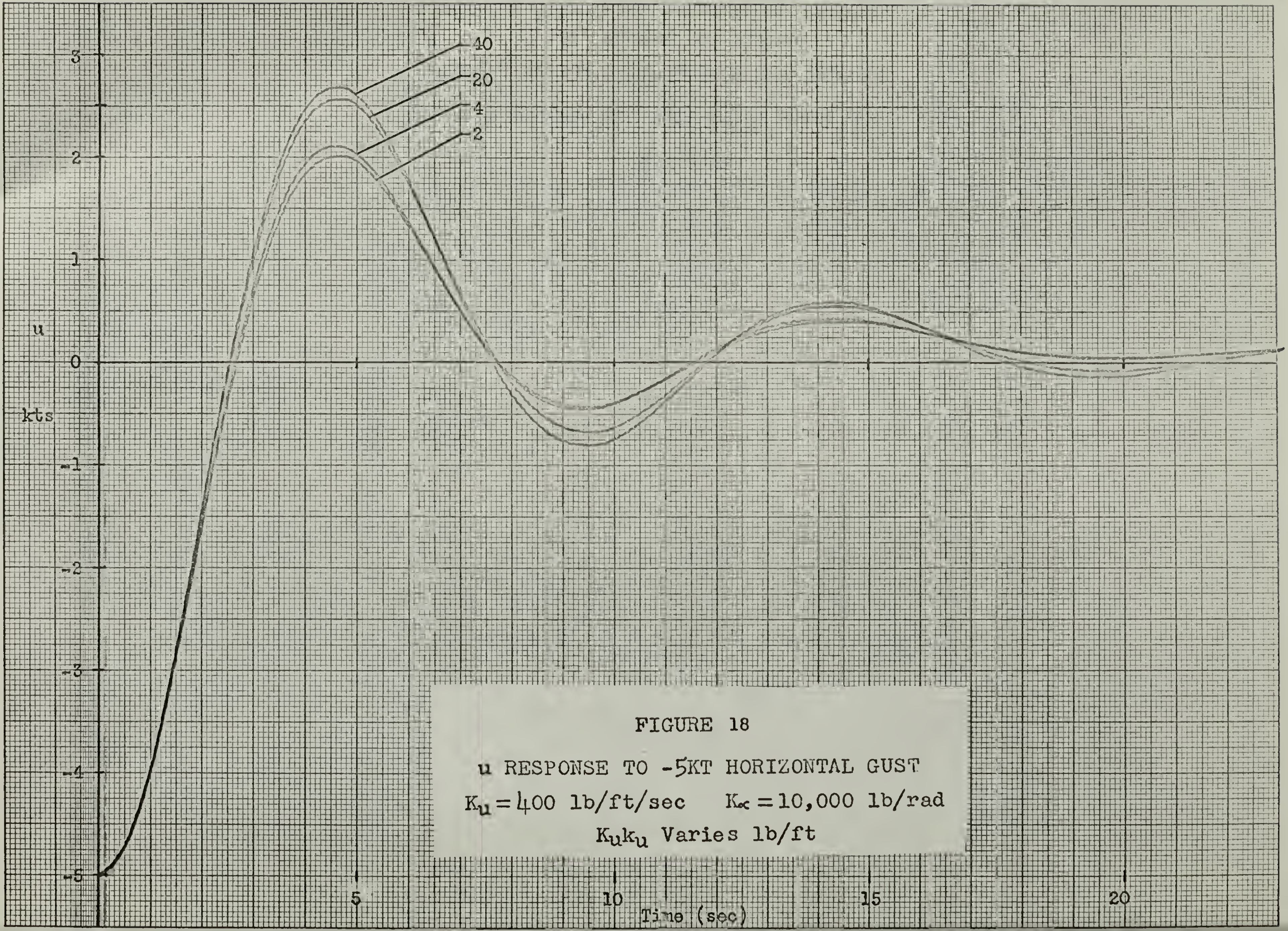


FIGURE 18

u RESPONSE TO -5KT HORIZONTAL GUST

$K_u = 400$ lb/ft/sec $K_\alpha = 10,000$ lb/rad

$K_u k_u$ Varies lb/ft

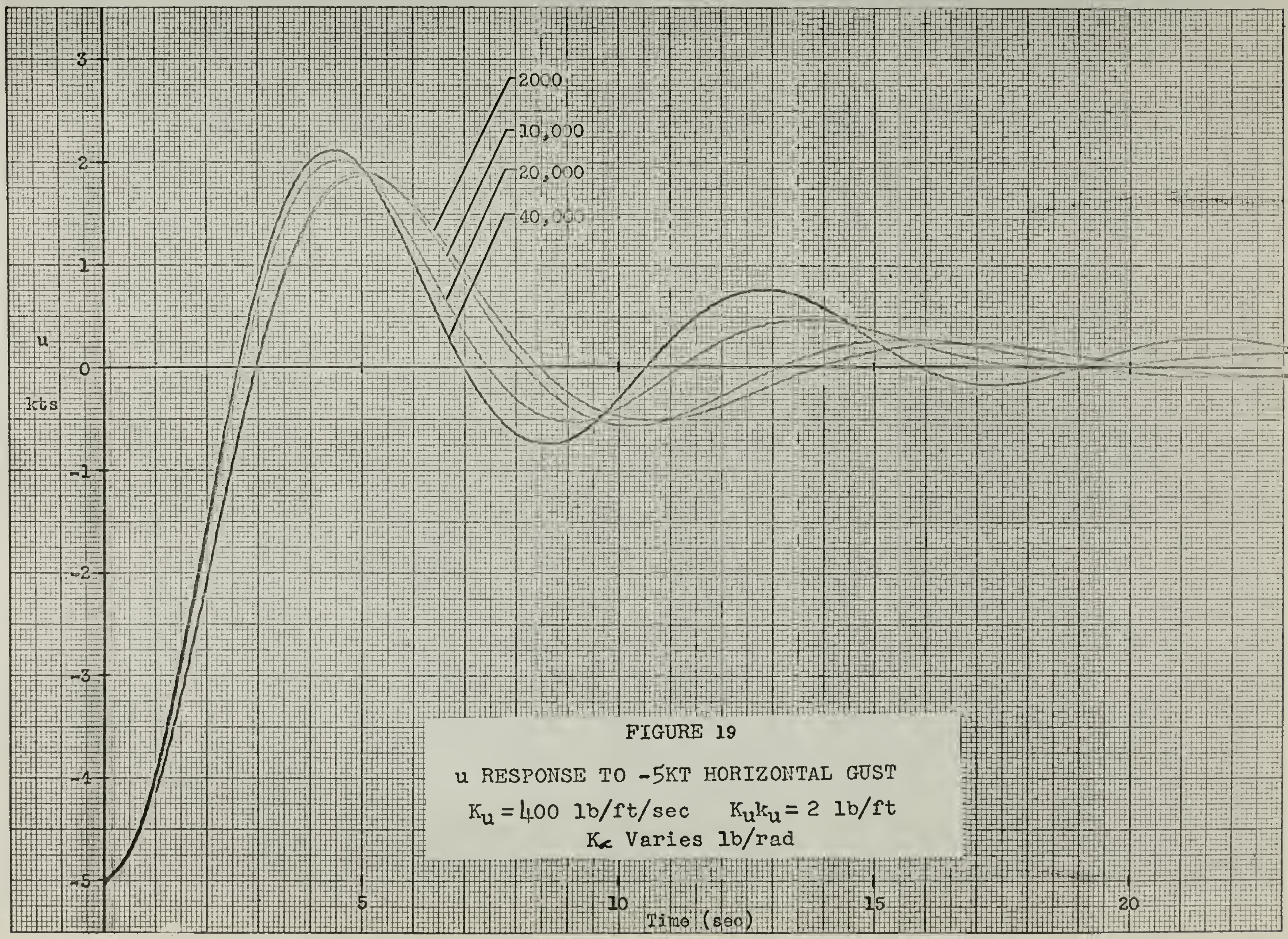


FIGURE 19

u RESPONSE TO -5KT HORIZONTAL GUST

$K_u = 400$ lb/ft/sec $\quad K_u k_u = 2$ lb/ft

K_α Varies lb/rad

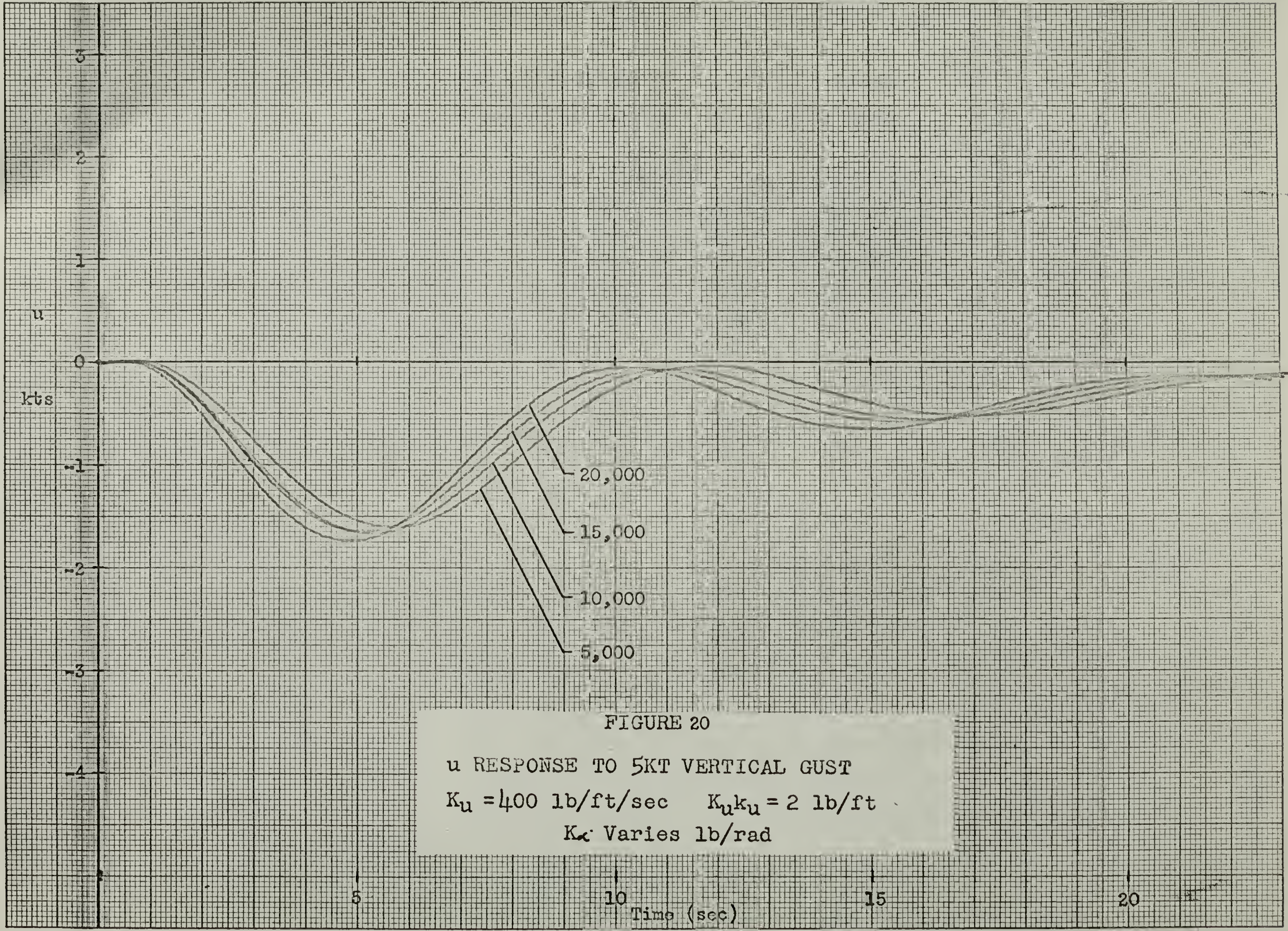


FIGURE 20

u RESPONSE TO 5KT VERTICAL GUST

K_u = 400 lb/ft/sec $K_u k_u$ = 2 lb/ft

K_α Varies lb/rad

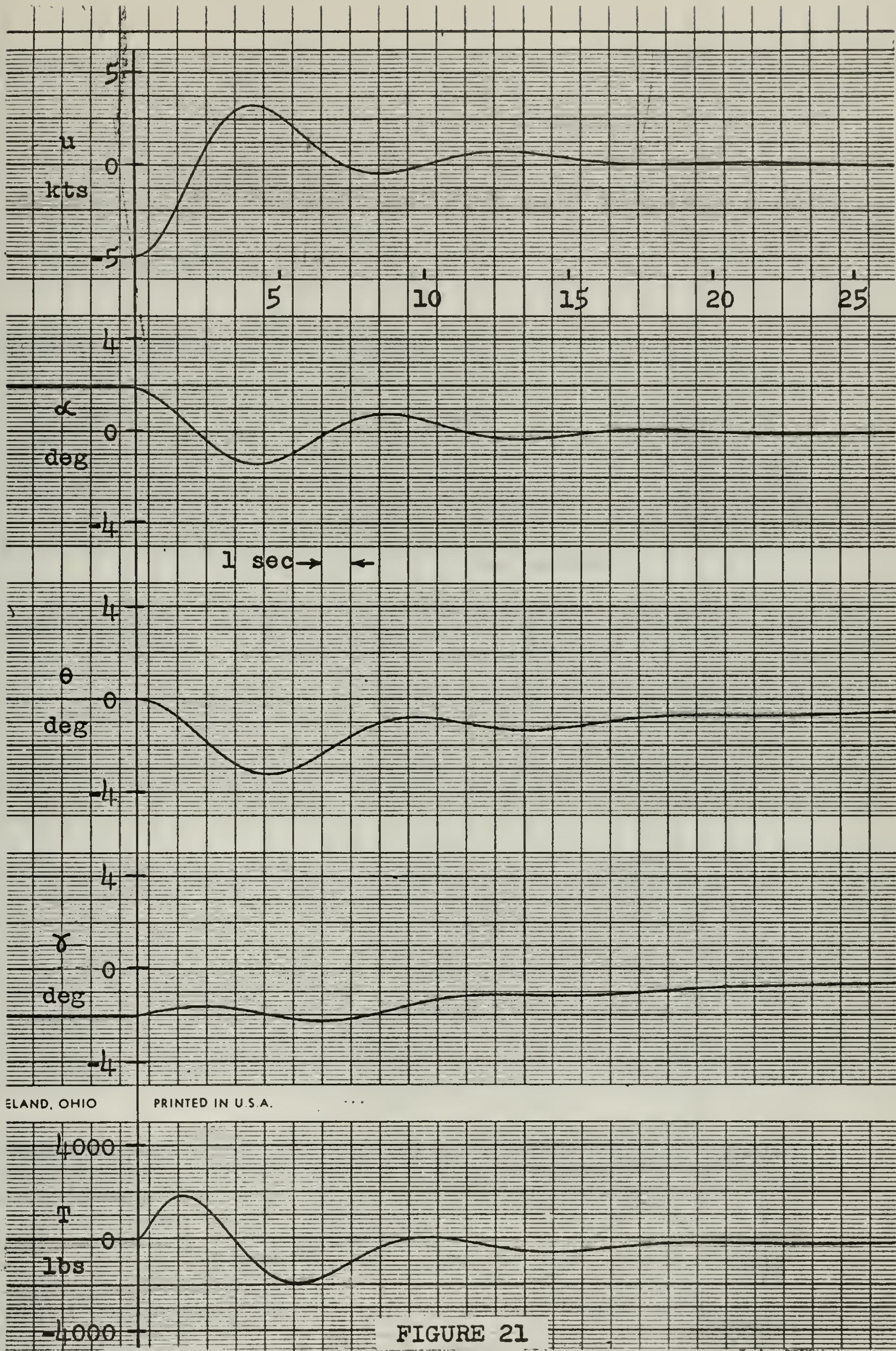


FIGURE 21

Airframe and Power Compensator -5 kts Horizontal and +5 kts($\alpha = 2.1°$) Vertical Gust Inputs

$K_u = 400 \quad K_u k_u = 2 \quad K_\alpha = 10{,}000$

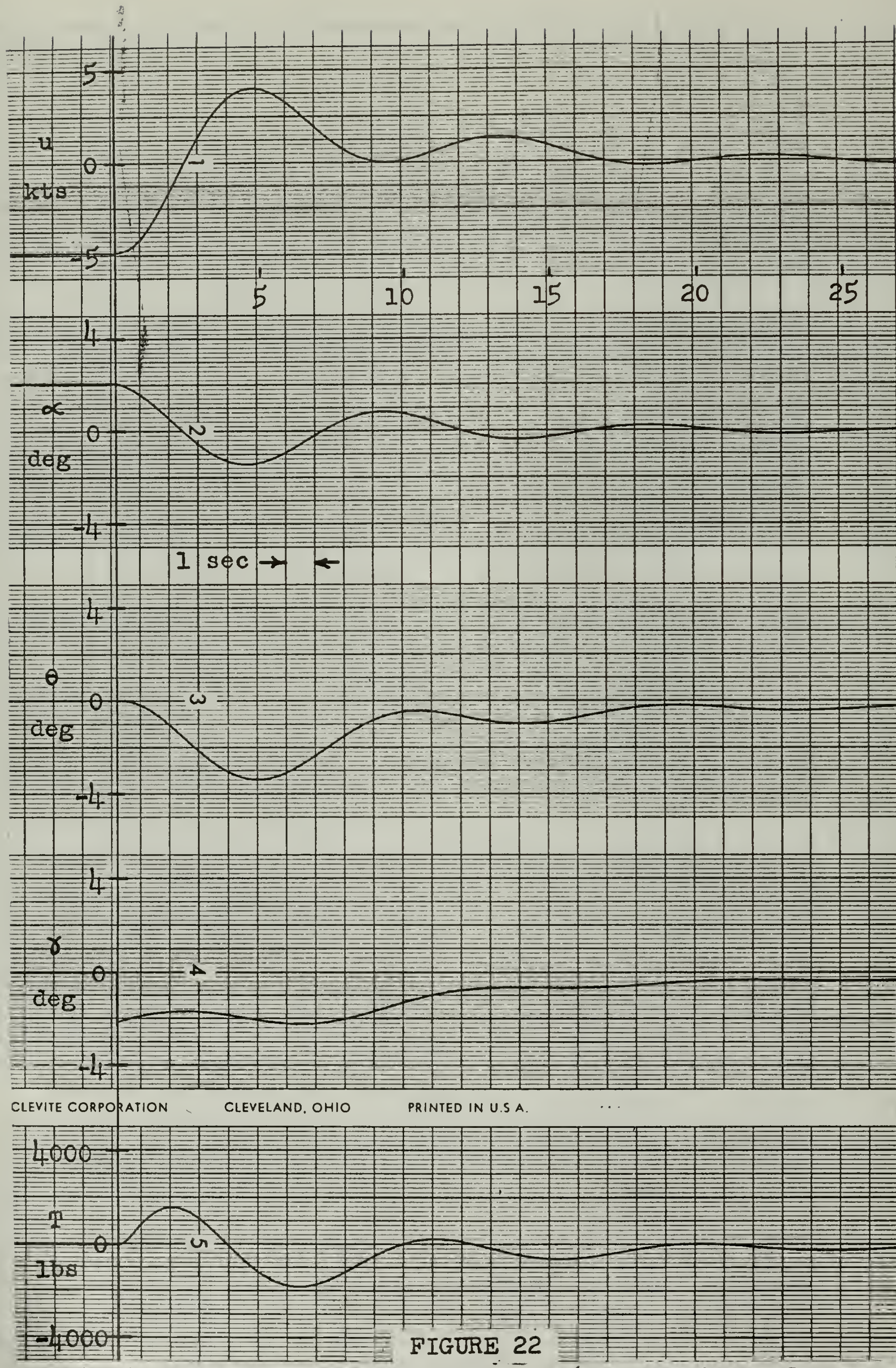


FIGURE 22

Airframe and Power Compensator -5 kts Horizontal and +5 kts(α = 2.1°) Vertical Gust Inputs

$K_u = 400 \quad K_u k_u = 2 \quad K_\alpha = 40{,}000$

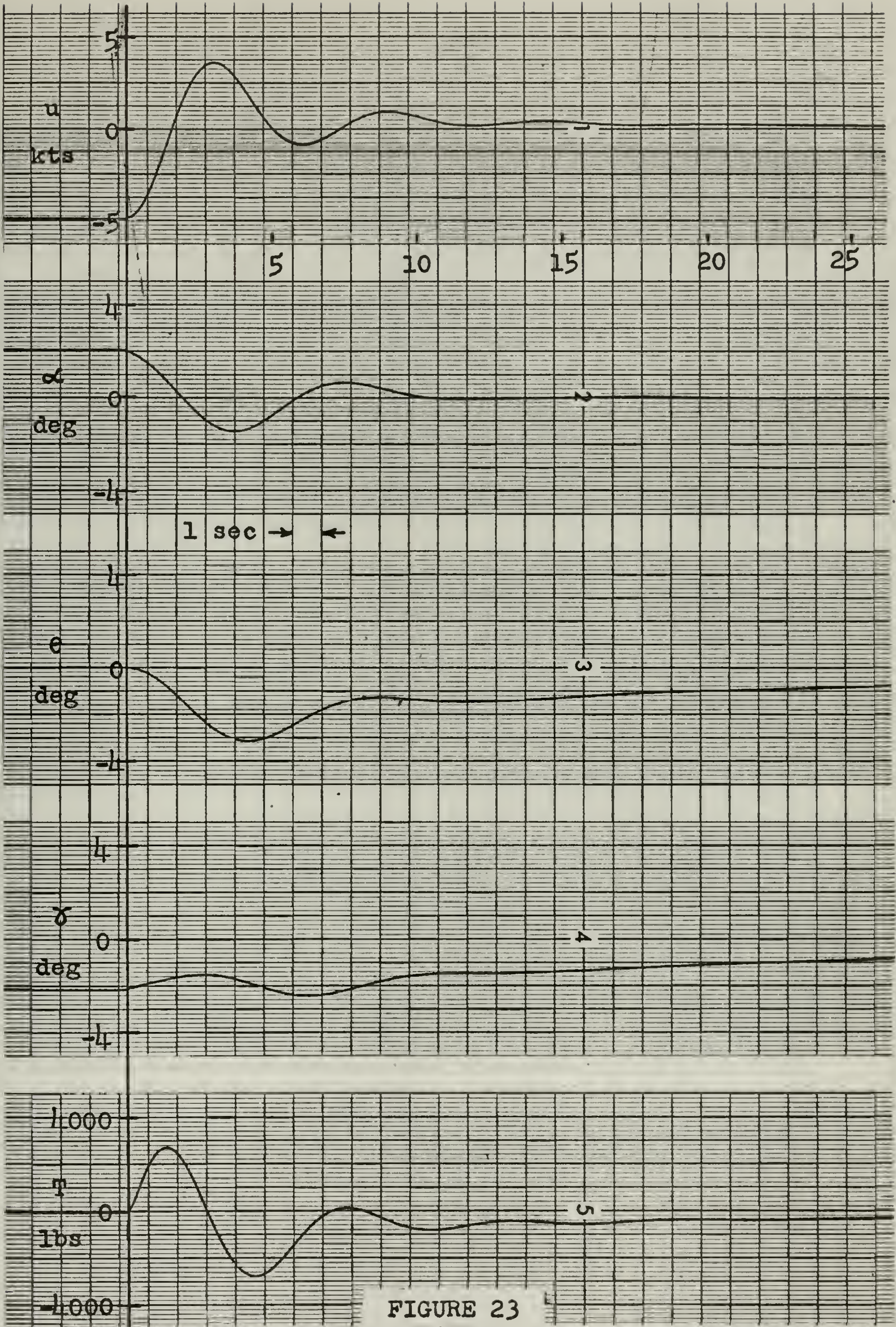


FIGURE 23
Airframe and Power Compensator -5 kts Horizontal and +5 kts(α=2.1°) Vertical Gust Inputs
K_u= 600 $K_u k_u$= 2 K_α=10,000

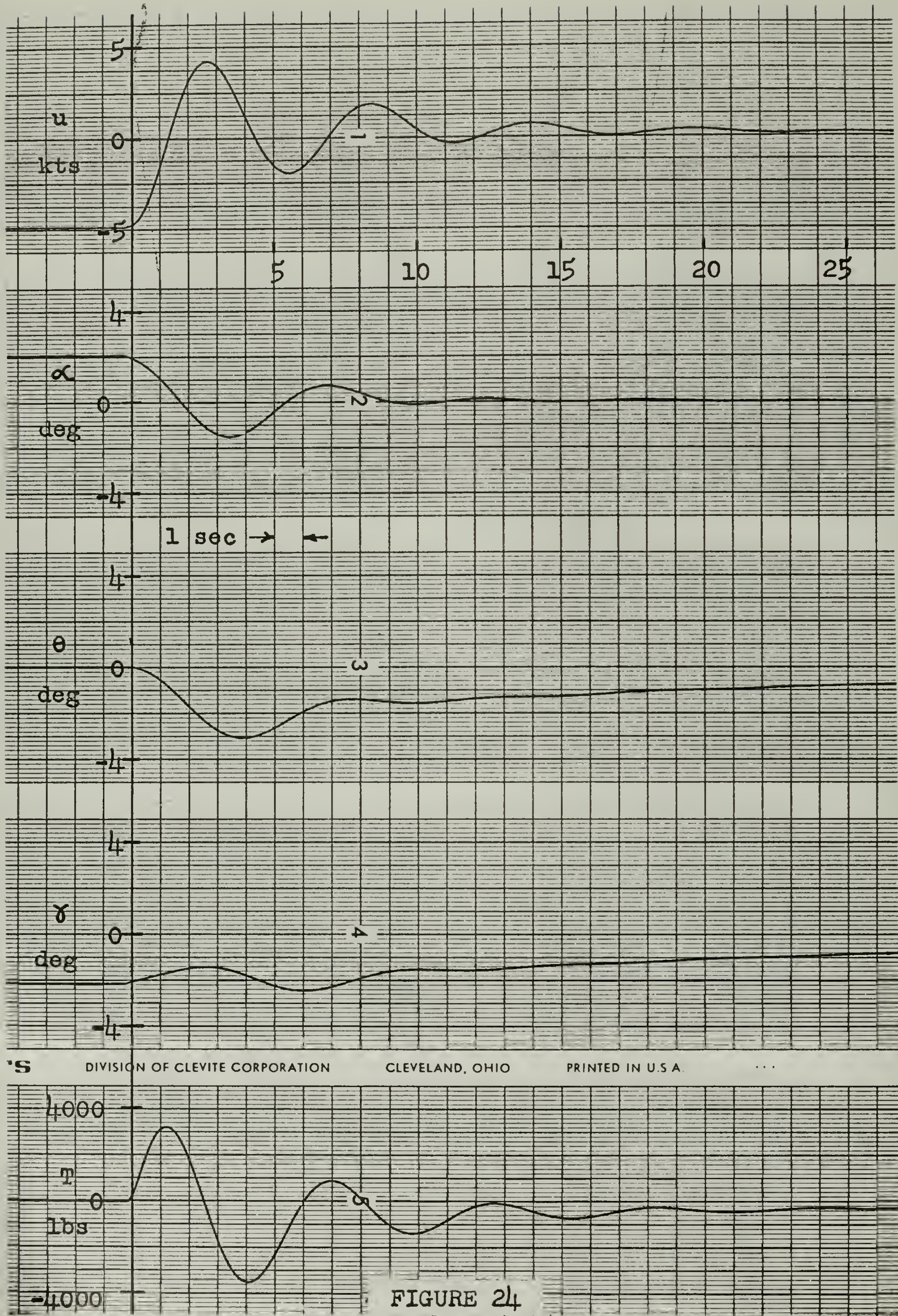


FIGURE 24

Airframe and Power Compensator -5 kts Horizontal and +5 kts(α=2.1°) Vertical Gust Inputs

$K_u = 600 \quad K_u k_u = .5 \quad K_\alpha = 40{,}000$

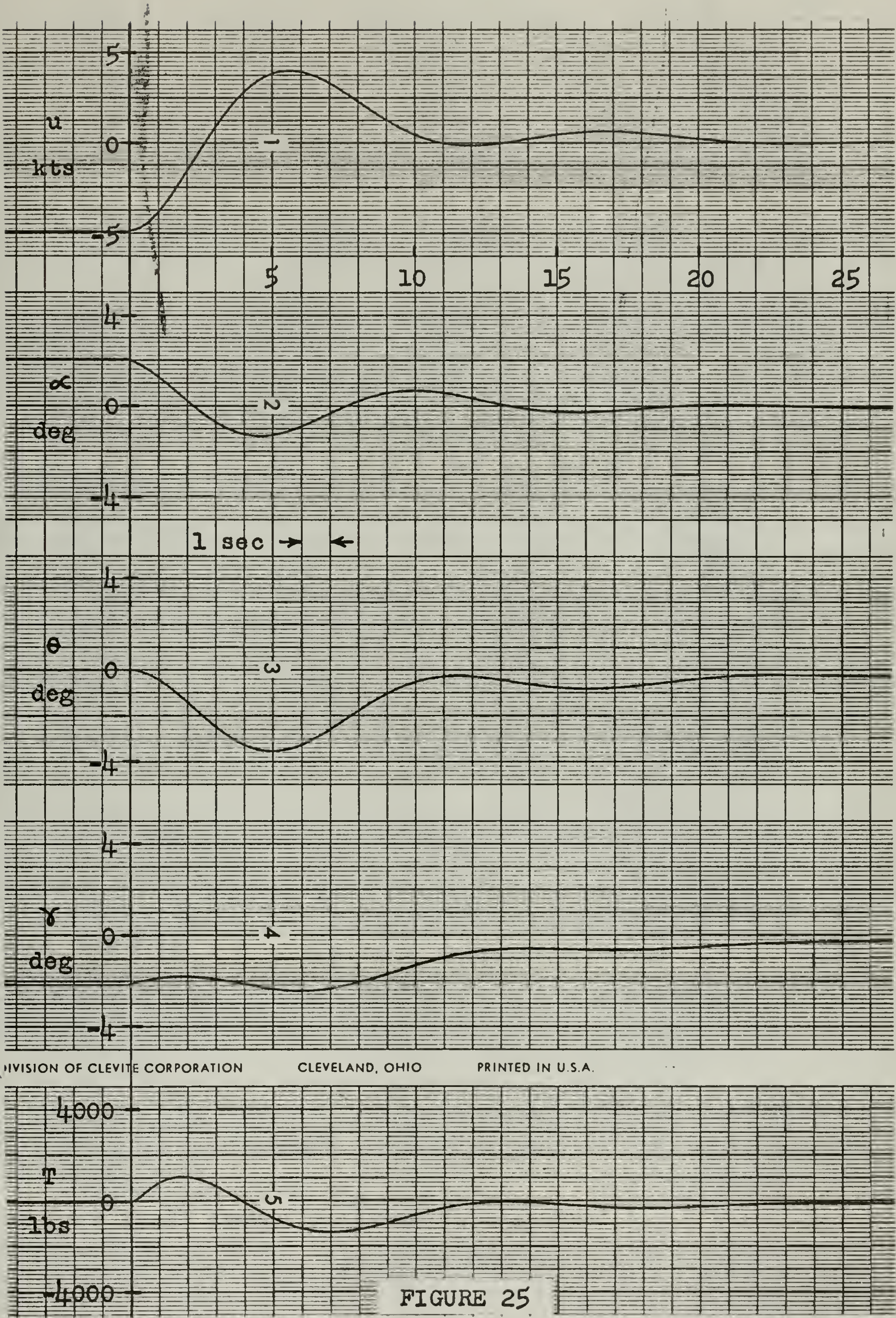


FIGURE 25

Airframe and Power Compensator -5 kts Horizontal and +5 kts($\alpha = 2.1^\circ$) Vertical Gust Inputs
$K_u = 200$ $K_u k_u = 2$ $K_\alpha = 10{,}000$

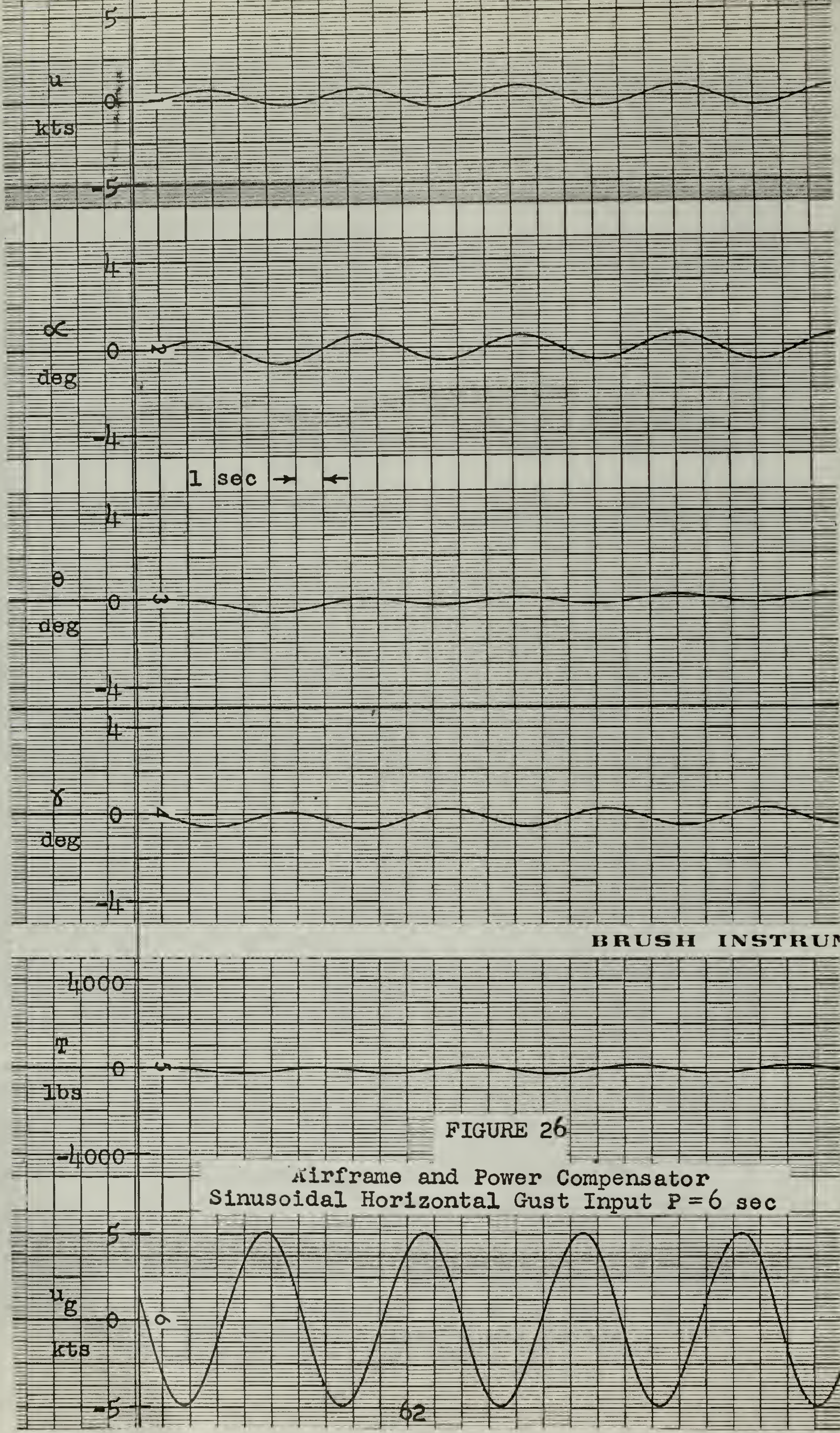


FIGURE 26

Airframe and Power Compensator
Sinusoidal Horizontal Gust Input P = 6 sec

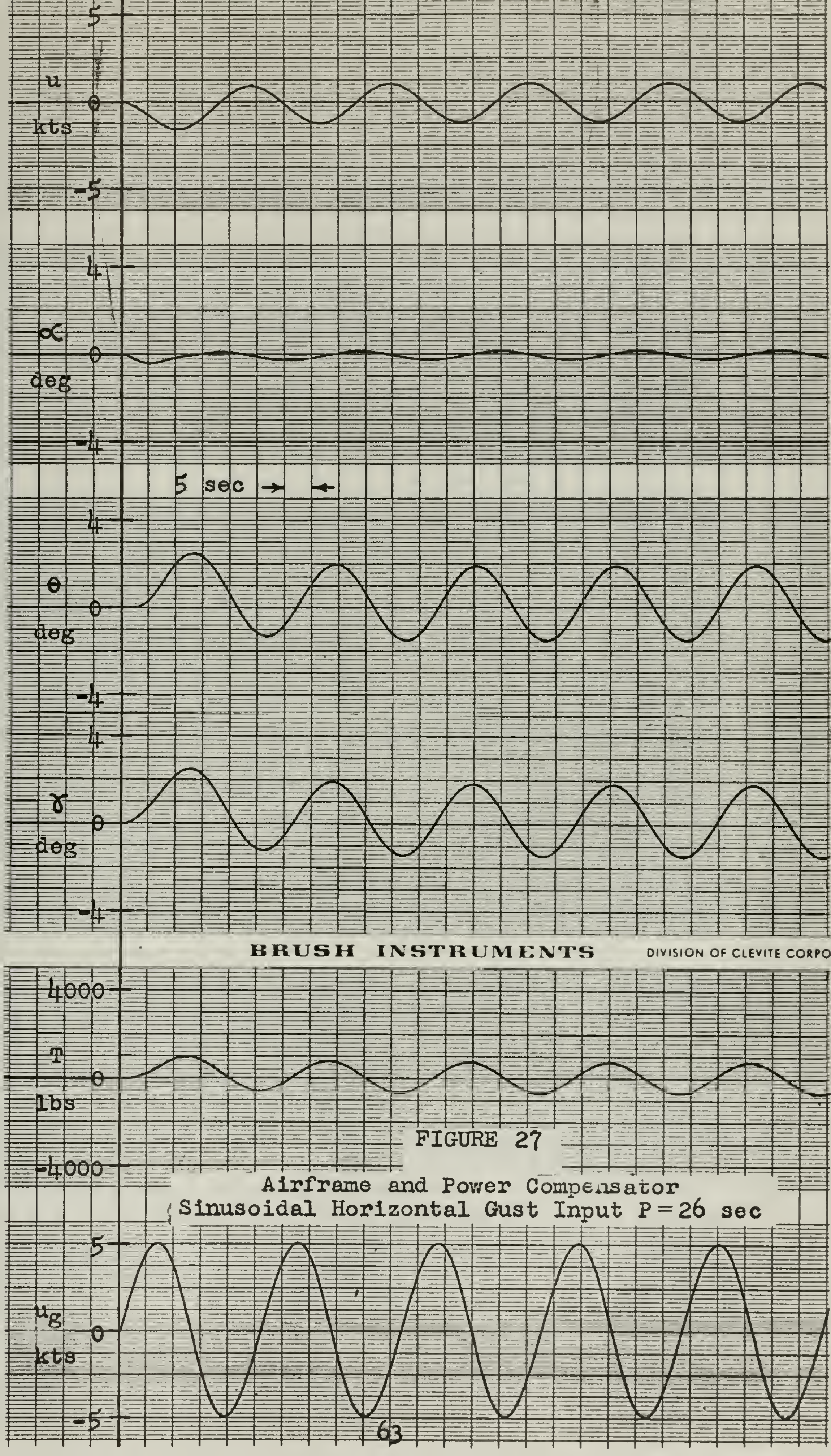


FIGURE 27

Airframe and Power Compensator
Sinusoidal Horizontal Gust Input P = 26 sec

FIGURE 28

Airframe and Power Compensator
Sinusoidal Vertical Gust Input P=6 sec

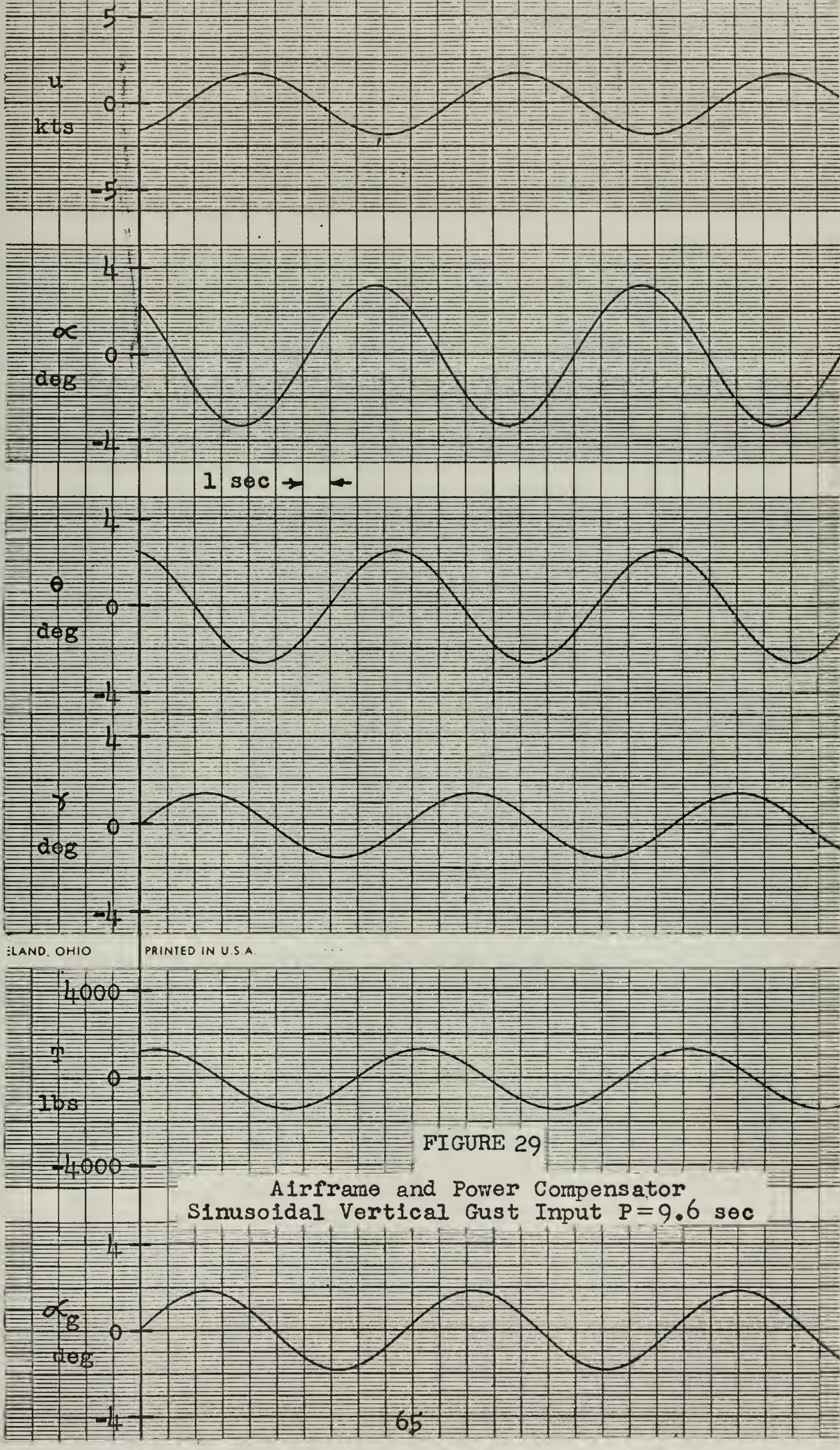


FIGURE 29

Airframe and Power Compensator
Sinusoidal Vertical Gust Input P=9.6 sec

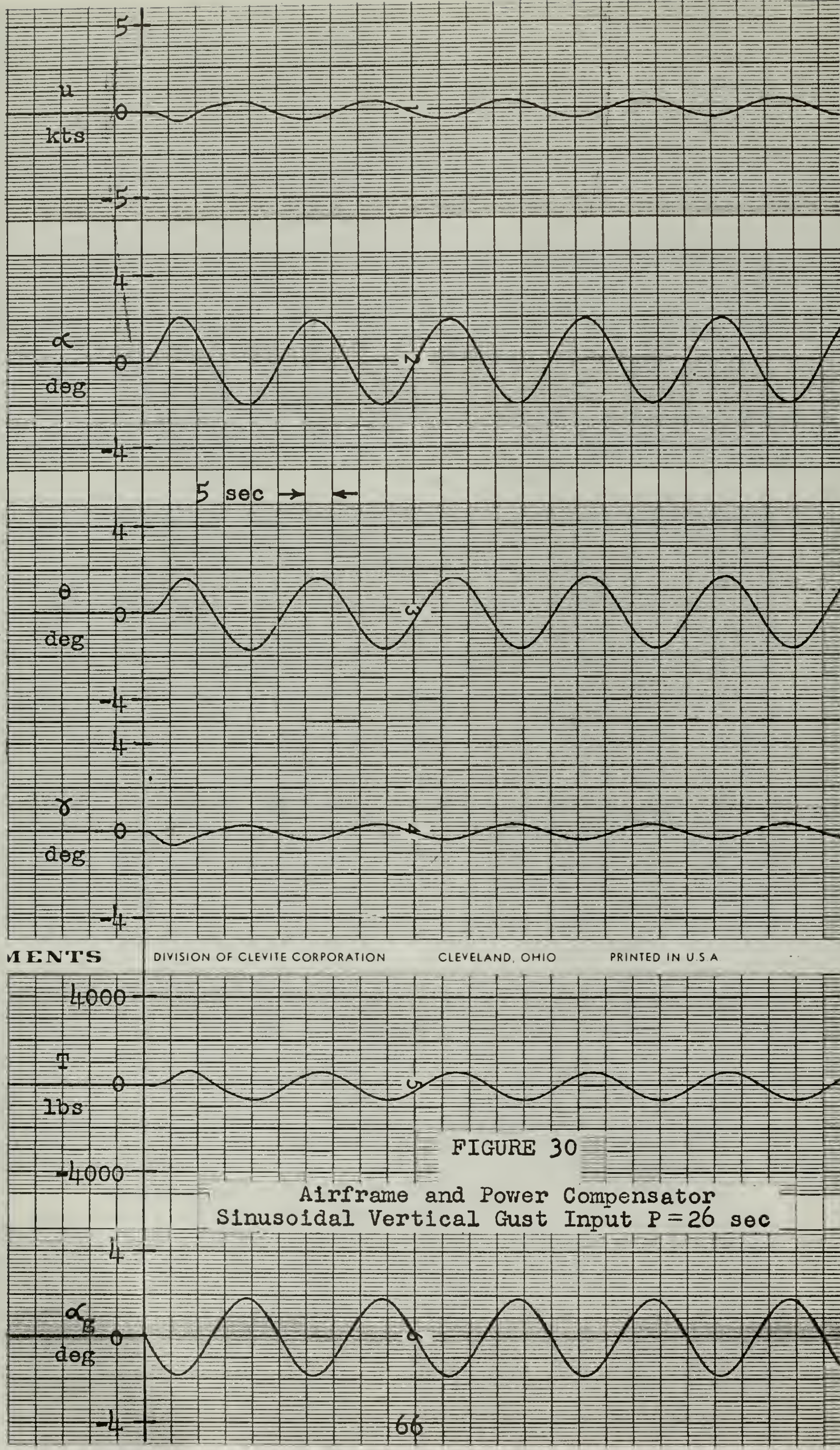


FIGURE 30

Airframe and Power Compensator
Sinusoidal Vertical Gust Input P = 26 sec

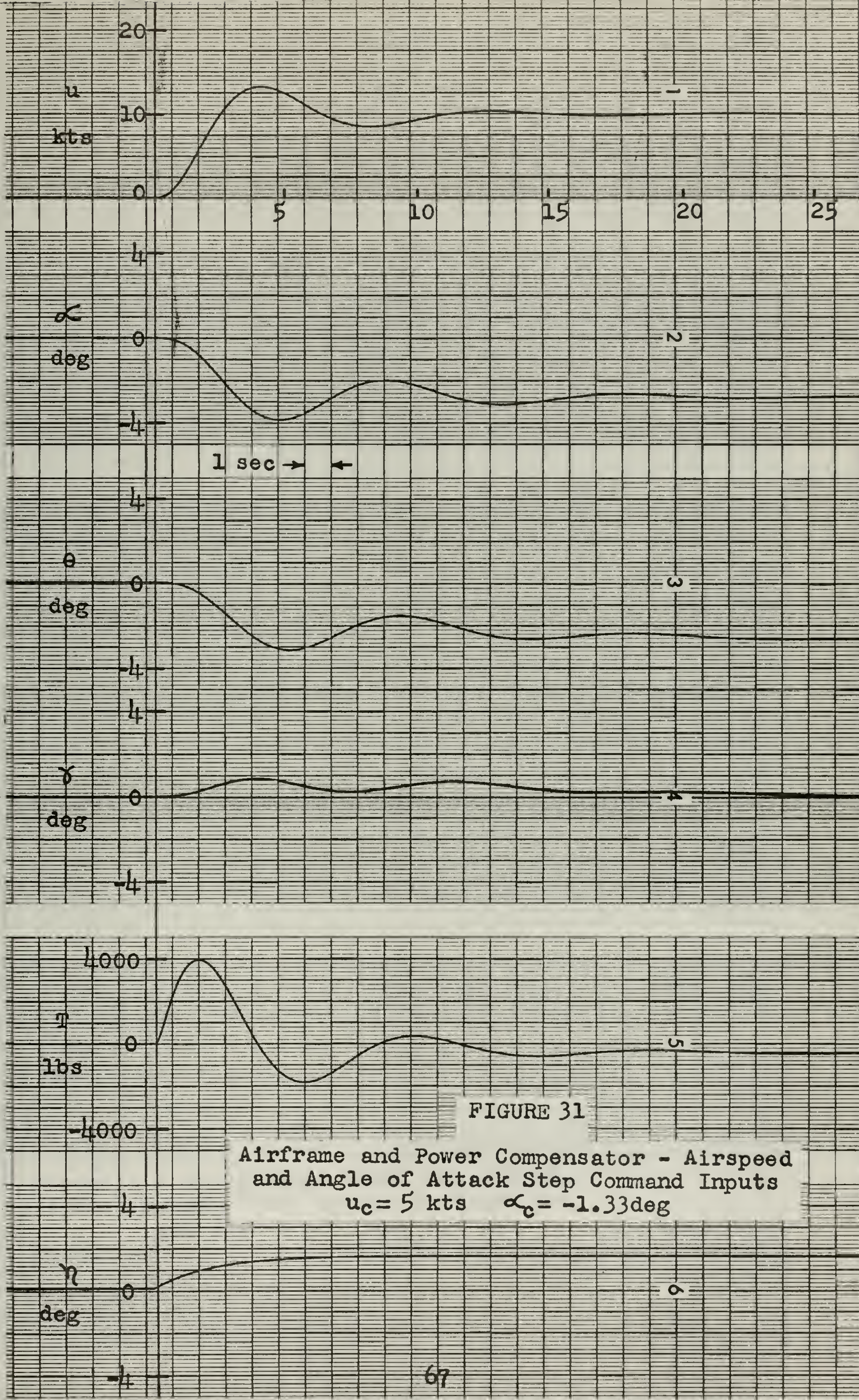


FIGURE 31

Airframe and Power Compensator - Airspeed and Angle of Attack Step Command Inputs
$u_c = 5$ kts $\alpha_c = -1.33$deg

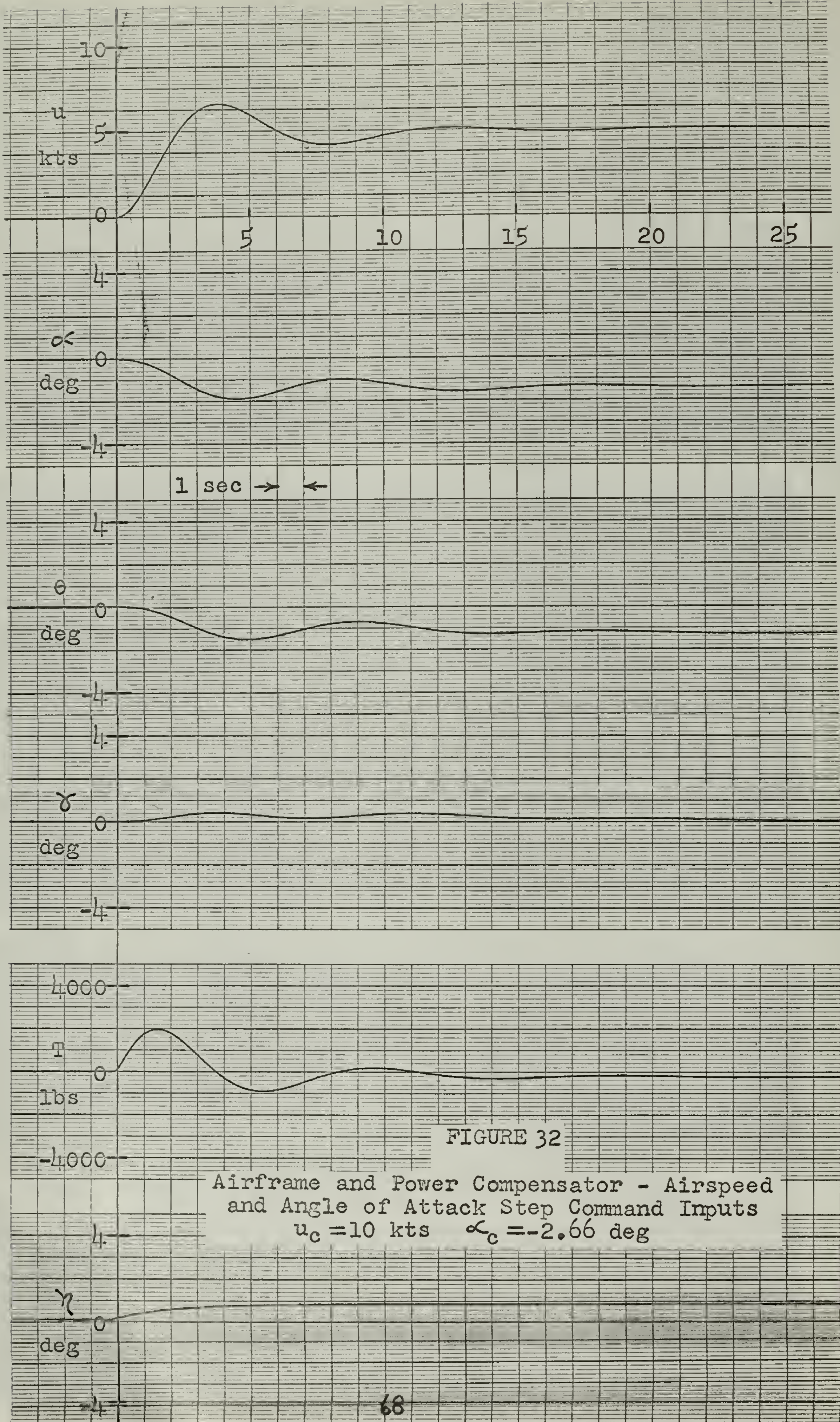


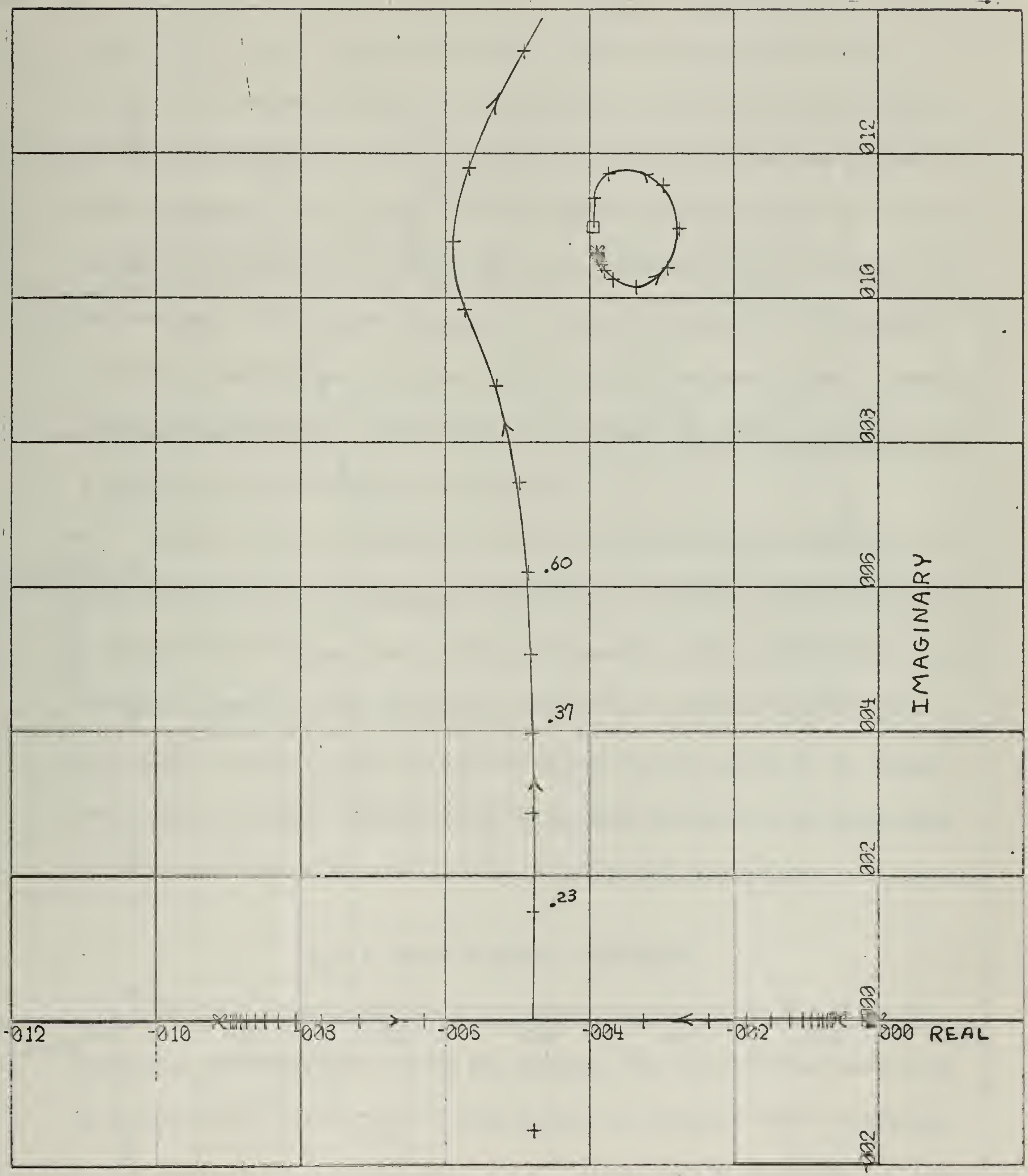
FIGURE 32

Airframe and Power Compensator - Airspeed and Angle of Attack Step Command Inputs
$u_c = 10$ kts $\alpha_c = -2.66$ deg

FIGURE 33

ROOT LOCUS
Complete Characteristic Equation

$K_u = 600 \quad K_\alpha = 10{,}000 \quad k_u$ Varies

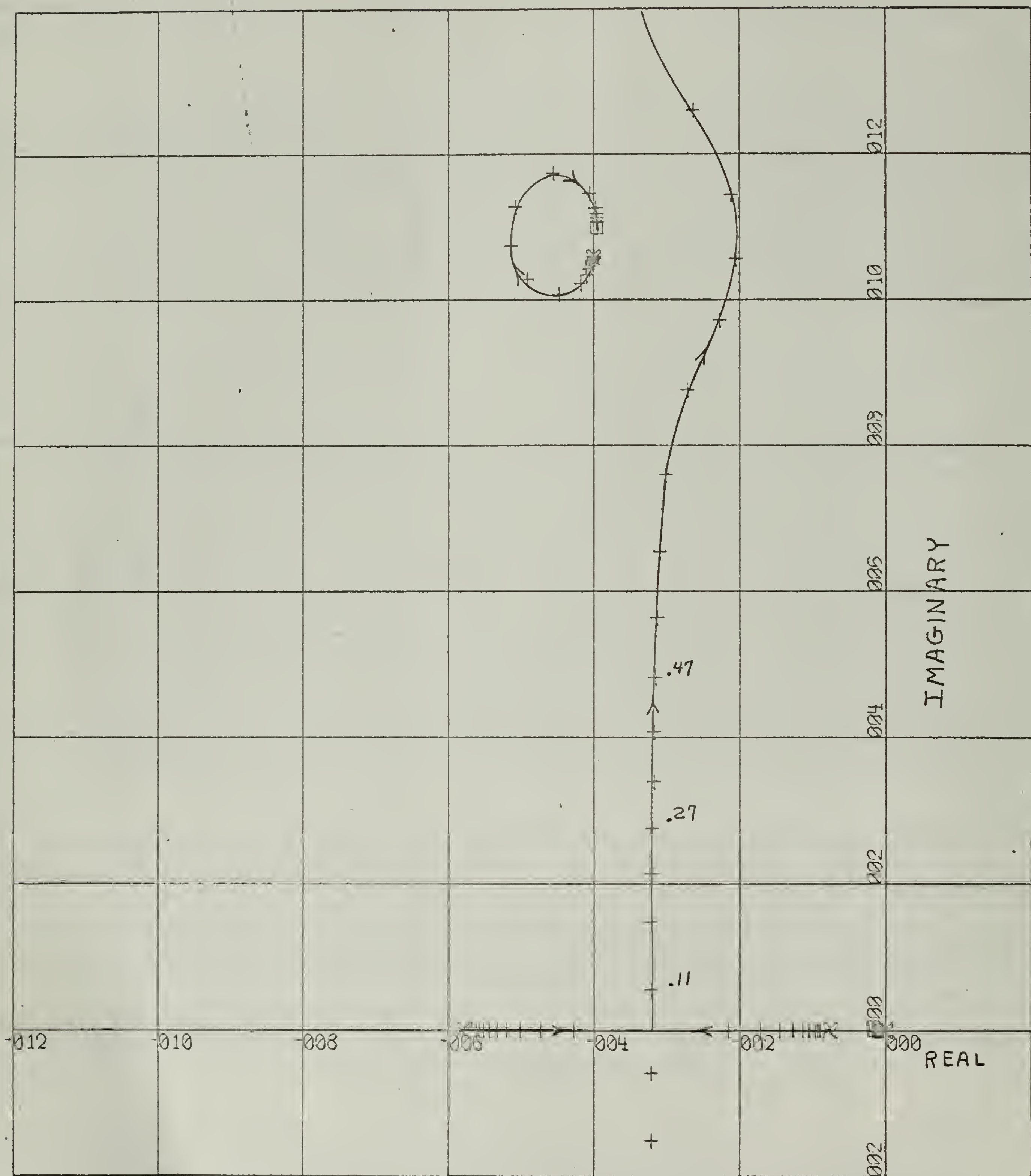


X-SCALE = 2.00E-01 UNITS/INCH.
Y-SCALE = 2.00E-01 UNITS/INCH.

FIGURE 34

ROOT LOCUS
Complete Characteristic Equation

$K_u = 400 \quad K_\alpha = 10{,}000 \quad k_u$ Varies



X-SCALE = 2.00E-01 UNITS/INCH.
Y-SCALE = 2.00E-01 UNITS/INCH.

7. APPENDIX

7.1 Digital Computer Programs

The digital computer programs are designed to compute the characteristic equation and the transfer functions for elevator and thrust inputs. By utilizing the proper combination of subprograms either wind tunnel data and airframe dimensions or reported nondimensional derivatives may be used in these programs. The subprograms developed here include only the longitudinal motion; however, any number of other subprograms may be added without revision. The program language is defined in Table VI. All input data must be in the foot-pound-radian system unless otherwise noted. Standard programing procedures for FORTRAN 63 are used. [1] The subprograms and a sample print out are shown in Figure 35.

Several of the subprograms require the use of another subprogram to compute the roots of the generated polynomials. These subroutines are of a standard type but the usage will vary depending on the installation. The subroutine used in these programs to compute the roots is RTPLSUB which is a library routine available at the Computer Facility of the U. S. Naval Postgraduate School. However, any subroutine designed to compute roots of polynomials could be used by rewritting the call statements.

7.1.1 Main Program - AEROFUN

The main program may be of any desired format so as to utilize the necessary subprograms and print the results. The main program used here is dimensioned to take up to 50 sets of data for different flight conditions.

The variable NN is used to indicate the number of sets of data for analysis.

7.1.2 Subprogram COMPTDER

This subprogram computes those aerodynamic derivatives which must normally be approximated from the wind tunnel data. The equations utilized for this program are listed in Table VII below. [5]

TABLE VII

EQUATIONS UTILIZED IN COMPUTER PROGRAM COMPTDER

$$\frac{\delta\epsilon}{\delta\alpha} = \frac{27 \cos^{3/2}\Lambda \, C_{L\alpha}}{\left(.525 + .475\gamma^{.736}\right) AR^{.725} \left(\ell_t/c\right)^{.2} e^{.385(h_t/c)}}$$

$$C_{Lq_w} = .44 + 2\left(x_{ac} - x_{cg}\right) C_{L\alpha} \qquad C_{Lq_t} = 2\frac{S_t}{S_w}\frac{\ell_t}{c}\eta_t C_{L\alpha_t}$$

$$C_{mq_w} = -C_{Lq_w}\left(x_{ac} - x_{cg}\right) - C_{L\alpha}\left[\frac{AR + 2\cos\Lambda}{8\,AR} + \left(\frac{AR^2 \tan^2\Lambda}{24}\right)\left(\frac{AR + 2\cos\Lambda}{AR + 6\cos\Lambda}\right)\right] \quad [10]$$

$$C_{mq_t} = C_{Lq_t}\frac{\ell_t}{c} \qquad C_{mq_B} = -.04\left(\frac{S_B}{S_w}\right)\left(\frac{\ell_B}{c}\right)^2$$

$$C_{Lq} = C_{Lq_w} + C_{Lq_t} \qquad C_{mq} = C_{mq_w} + C_{mq_t} + C_{mq_B}$$

$$C_{L\delta e} = -\frac{S_t}{S_w}\eta_t \tau C_{L\alpha_t}$$

$$C_{m\dot{\alpha}} = -2\frac{S_t}{S_w}\left(\frac{\ell_t}{c}\right)^2 \eta_t \frac{\delta\epsilon}{\delta\alpha}$$

$$C_{m\delta e} = -\frac{S_t}{S_w}\frac{\ell_t}{c}\eta_t \tau C_{L\alpha_t} \qquad C_{m\dot{\delta} e} = \frac{S_t}{S_w}\left(A + B C_{L\alpha_t}\right)$$

INPUTS: Λ, $C_{L\alpha}$, $C_{L\alpha_t}$, AR, c, S_w, h_t, η_t, S_t, τ, A, B, x_{ac}, x_{cg}, l_t, $\frac{\partial\epsilon}{\partial\alpha}$.

NOTE: If $C_{L\alpha_t}$ is based on the wing reference area, assign S_w and S_t values of one.

$\frac{\partial\epsilon}{\partial\alpha}$ may either be an input or it may be computed in the program. If it is to be an input, assign the value to DEDAI. If it is to be computed, assign zero to DEDAI.

OUTPUT: C_{Lq}, $C_{L\delta e}$, C_{mq}, $C_{m\delta e}$, $C_{m\dot{\delta} e}$, $C_{m\alpha}$, $\frac{\partial\epsilon}{\partial\alpha}$.

7.1.3 Subprogram ENGDIMEN

This subprogram computes the English dimensional derivatives from the NACA non dimensional derivatives. The relationships used in this subprogram are given in Table II.

INPUTS: U_o, ρ, W, S_w, B, c, μ, z_T, l_t, C_L, $C_{L\alpha}$, C_{Lu}, C_{Lq}, $C_{L\delta e}$, C_D, $C_{D\alpha}$, C_{Du}, C_{Dq}, $C_{D\delta e}$, $C_{m\alpha}$, $C_{m\dot{\alpha}}$, C_{mq}, $C_{m\delta e}$, C_T, θ_o.

OUTPUTS: X_u, X_w, X_q, X_η, X_T, Z_u, Z_w, Z_q, Z_η, Z_T, M_u, M_w, $M_{\dot{w}}$, M_q, M_η, M_T, $g \cos\theta_o$, $g \sin\theta_o$.

7.1.4 Subprogram STAB

This subroutine computes the coefficients of the characteristic equation, the roots of the equation and the characteristics of the short period and long period motions. The equations used in this program are given in 7.2.

INPUTS: U_o, θ_o, X_u, X_w, Z_u, Z_w, Z_q, M_u, M_w, $M_{\dot{w}}$, M_q.

OUTPUTS: (as defined in Table VI) U, V, CONV, WNLP, WNSP, ZLP, ZSP, TLP, THLP, THSP, TTLP, TTSP.

NOTES: Requires a subprogram to evaluate the roots of the generated polynominal. The coefficients of polynominal are stored in the dimensioned variable C. The order of the polynomial is assigned to the variable N. The outputs of the called subroutine (the roots of the polynominal) are assigned to U and V and are therefore preserved for subsequent calculations.

7.1.5 Subprogram FORCEFUN

This subprogram computes and prints the coefficients and the roots of the polynominal numerators of the transfer functions α/η, u/η, α/T, u/T. The equations utilized in the program are given in 7.2.

INPUTS: U_o, θ_o, X_u, X_w, X_T, Z_u, Z_w, Z_q, Z_η, M_u, M_w, $M_{\dot{w}}$, M_q, M_T, M_η .

OUTPUTS: None.

NOTES: Requires a subprogram to evaluate the roots of the generated polynomial. The operation is the same as for STAB except the output is printed internally.

7.2 Derivation of Airframe Transfer Functions

The airframe transfer functions are derived from the equations of motion written in Laplace Matrix form given previously as Eqs 2-6. The

TABLE VI

DEFINITION OF SYMBOLS

Program Mnenonic	Parameter
AR	AR Aspect Ratio
B	B (I_{yy}, slug-ft^2)
C, CH	c chord
CD	C_D
CDA	$C_{D\alpha}$
CDDE	$C_{D\delta e}$
CDQ	C_{Dq}
CDU	C_{Du}
CL	C_L
CLA	$C_{L\alpha}$
CLAT	$C_{L\alpha_t}$
CLDE	$C_{L\delta e}$
CLQ	C_{Lq}
CLU	C_{Lu}
CMA	$C_{m\alpha}$
CMAD	$C_{m\dot{\alpha}}$
CMDD	$C_{m\dot{\delta}e}$
CMDE	$C_{m\delta e}$
CMQ	C_{mq}
CT	C_T
CONV	Convergence indicator for roots output of STAB only
D	z_T Perpendicular distance between the cg and thrust axis. Positive for thrust axis above cg.

DEDA,DEDAI	$\frac{\partial \epsilon}{\partial \alpha}$
ET	η_t effectiveness of horizontal tail
GCTHETA	g $\cos\theta$
GSTHETA	g $\sin\theta$
HT	h_t height of root chord position of horizontal tail above wing root chord.
NN	Number of sets of data - Main Program only
R	ρ (slugs/ft^3)
ST	S_t
SW,S	S_w or reference area
TAU	τ $\frac{\partial \alpha_t}{\partial \delta_e}$
THETAD	θ_o (deg)
TLP	Period of Phugoid Motion
TSP	Period of Short Period Motion
THLP	Phugoid time to $\frac{1}{2}$
THSP	Short Period tim to $\frac{1}{2}$
TTLP	Phugoid time to .1
TTSP	Short Period time to .1
TR	λ taper ratio
U,UO	U_o reference or steady state airspeed
U	Output of real parts of roots STAB only
V	Output of imaginary parts of roots - STAB only
W	weight
WNLP	w_n phugoid
WNSP	w_n short period

TABLE VI (CONT)

XETA	X_η	
XLAMBDA	Λ	sweep (deg)
XL	l_t	distance from cg to aerodynamic center of horizontal tail
XMU	M_u	
XMETA	M_η	
XMQ	M_q	
XMT	M_T	
XMW	M_w	
XMWD	$M_{\dot{w}}$	
XU	X_u	
XQ	X_q	
XT	X_T	
XW	X_w	
ZE	ν	angle between thrust line and body axis, positive if upward (deg)
ZET	Z_η	
ZQ	Z_q	
ZT	Z_T	
ZU	Z_u	
ZW	Z_w	
ZLP	ζ	of phugoid
ZSP	ζ	of short period
A & B	COMPTDER only - values from Perkins & Hage, Fig 10-10.	
XAC	x_{ac}	position of aerodynamic center of wing in % MAC
XCG	x_{cg}	position of center of gravity in % MAC

FIGURE 35

DIGITAL COMPUTER PROGRAMS

```
      PROGRAM AEROFUN
C                 ****
C     THIS PROGRAM IS DESIGNED TO COMPUTE THE CHARACTERISTIC EQUATION
C     AND THE NUMERATOR POLYNOMIALS FOR THE TRANSFER FUNCTIONS U/T,
C     ALPHA/T, U/ETA, AND ALPHA/ETA FOR ONE SET OF NACA DERIVATIVES
C     CL=.87     CLA=2.598   CLU=0    CLQ=-2.82      CLDE=.539
C     CD=.197    CDA=.963    CDU=0    CDQ=0          CDDE=-.046
C     CMA=-.38  CMAD=-.55    CMQ=-4.5   CMDE=-.752   CT=.195
C                 ****
      DIMENSION U(50),RHO(50),W(50),THETAD(50),ALPHA(50),CL(50),CLA(50),
     1CLU(50),CLQ(50),CLDE (50),CD(50),CDA(50),CDU(50),CDQ(50),
     2CDDE (50),CMA(50),CMAD(50),CMQ(50),CMDE (50),CT(50),C(5),RR(4),
     3RI(4),CONV(4)
      READ 1,S,B,CH,XL,D,ZE   ,NN
    1 FORMAT(6F10.0,I5)
      READ 2,(U(I),RHO(I),W(I),THETAD(I),ALPHA(I),CL(I),CLA(I),CLU(I),
     1CLQ(I),CLDE (I),CD(I),CDA(I),CDU(I),CDQ(I),CDDE (I),CMA(I),
     2CMAD(I),CMQ(I),CMDE (I),CT(I),I=1,NN)
    2 FORMAT(5F10.0 /5F10.0/5F10.0/5F10.0)
      DO 200 J=1,NN
      PRINT 6
    6 FORMAT(1H1)
      CALL ENGDIMEN(U(J),RHO(J),W(J),S,B,CH,ZE   ,D,XL,CL(J),CLA(J),
     1 CLU(J),CLQ(J),CLDE (J),CD(J),CDA(J),CDU(J),CDQ(J),CDDE (J),
     2CMA(J),CMAD(J),CMQ(J),CMDE (J),CT(J),THETAD(J),XU,XW,XQ,XETA,
     3XT,ZU,ZW,ZQ,ZET,ZT,XMU,XMW,XMWD,XMQ,XMETA,XMT,GCTHETA,GSTHETA)
      PRINT 5,U(J),RHO(J),W(J),ALPHA(J),THETAD(J)
    5 FORMAT(//41X,18H FLIGHT CONDITIONS// 5X6HUSUBO= F5.0,7H FT/SEC,7X
     19H DENSITY=E13.5,12H SLUGS/CB FT,5X8H WEIGHT=F7.0,4H LBS//
     2 5X6HALPHA=,F6.2,4H DEG,10X6HTHETA=,F6.2,4H DEG     )
```

FIGURE 35 (CONT)

```
      PRINT 67
   67 FORMAT(5X,89H---------------------------------------------------------
     1----------------------------------     ,/)
      PRINT 7
    7 FORMAT(   28X44H ENGLISH DIMENSIONAL AERODYNAMIC DERIVATIVES//)
      PRINT 8,XU,ZU,XMU,XW,ZW,XMW,GCTHETA,GSTHETA,XMWD,XQ,ZQ,XMQ,XT,ZT,
     1XMT,XETA,ZET ,XMETA
    8 FORMAT( 5X3HXU=F7.3,6H 1/SEC,16X3HZU=F7.3,6H 1/SEC,16X3HMU=E12.3,
     19H 1/SEC-FT,// 5X3HXW=F7.3,6H 1/SEC,16X3HZW=F7.3,6H 1/SEC,16X
     23HMW=E12.3,9H 1/SEC-FT,// 5X,13HGCOS(THETA0)=F7.3,8H FT/SEC2,4X
     313HGSIN(THETA0)=F7.3,8H FT/SEC2,4X6HMWDOT=E12.3,5H 1/FT,// 5X
     43HXQ=E12.3,11H FT/SEC-RAD,6X3HZQ=E12.3,10H 1/SEC-RAD,7X3HMQ=F7.3,
     510H 1/SEC-RAD,// 5X,3HXT=E12.3,11H FT/SEC2-LB,6X3HZT=E12.3,
     611H FT/SEC2-LB,6X3HMT=E12.3,10H 1/SEC2-LB,// 5X,5HXETA=E12.3,
     712H FT/SEC2-RAD,3X5HZETA=F8.3,12H FT/SEC2-RAD,8X5HMETA=F7.3,
     811H 1/SEC2-RAD     )
      CALL STAB(U,THETAD,XU,XW,ZU,ZW,ZQ,XMU,XMW,XMWD,XMQ,C,RR,RI,CONV,
     1WNLP,WNSP,ZLP,ZSP,TLP,TSP,THLP,THSP,TTLP,TTSP)
      PRINT 67
      PRINT 63
   63 FORMAT(   30X32HCHARACTERISTIC EQUATION ANALYSIS//)
      PRINT 54,(C(K),K=2,5)
   54 FORMAT(     5X,59HCHARACTERISTIC EQUATION - S**4 + A*S**3 + B*S**2
     1 + C*S + D //,5X2HA=E12.5,5X2HB=E12.5,5X2HC=E12.5,5X2HD=E12.5,/)
      PRINT 56
   56 FORMAT(12X5HROOTS,10X9HREAL PART,7X14HIMAGINARY PART,9X
     111HCONVERGENCE/)
      PRINT 55,(RR(L),RI(L),CONV(L), L=1,4)
   55 FORMAT(17X,3E20.5)
      PRINT 65
   65 FORMAT( /5X25HSTABILITY CHARACTERISTICS,5X12HPHUGOID MODE,5X
     112HSHORT PERIOD,/ )
```

FIGURE 35 (CONT)

```
      PRINT 66,WNLP,WNSP,ZLP,ZSP,TLP,TSP,THLP,THSP,TTLP,TTSP
   66 FORMAT(24X8HOMEGA N=2(F10.4,9X)/27X5HZETA=2(F10.4,9X)/25X7HPERIOD=
     12(F10.4,9X)/21X11HTIME TO .5=2(F10.4,9X)/21X11HTIME TO .1=2(F10.4,
     29X))
      CALL FORCEFUN(U,THETAD,XU,XW,XT,ZU,ZW,ZQ,ZET,XMU,XMW,XMWD,XMQ,
     1XMT,XMETA)
  200 CONTINUE
      END
```

FIGURE 35 (CONT)

FLIGHT CONDITIONS

USUBO= 234 FT/SEC	DENSITY= 2.37800E-03 SLUGS/CB FT	WEIGHT= 22000 LBS
ALPHA= 11.10 DEG	THETA= 8.10 DEG	

ENGLISH DIMENSIONAL AERODYNAMIC DERIVATIVES

XU= -.060 1/SEC	ZU= -.265 1/SEC	MU= 1.852E-04 1/SEC-FT
XW= -.014 1/SEC	ZW= -.426 1/SEC	MW= -4.865E-03 1/SEC-FT
GCOS(THETAO)= 31.853 FT/SEC2	GSIN(THETAO)= 4.533 FT/SEC2	MWDOT= -1.772E-04 1/FT
XQ= 0 FT/SEC-RAD	ZQ= -2.534E 00 1/SEC-RAD	MQ= -.339 1/SEC-RAD
XT= 1.462E-03 FT/SEC2-LB	ZT= -2.170E-05 FT/SEC2-LB	MT= -4.552E-06 1/SEC2-LB
XETA= -1.642E 00 FT/SEC2-RAD	ZETA= -19.245 FT/SEC2-RAD	META= -2.253 1/SEC2-RAD

CHARACTERISTIC EQUATION ANALYSIS

CHARACTERISTIC EQUATION - S**4 + A*S**3 + B*S**2 + C*S + D

A= 8.66955E-01 B= 1.31474E 00 C= 6.10246E-02 D= 4.23216E-02

ROOTS	REAL PART	IMAGINARY PART	CONVERGENCE
	-1.30085E-02	-1.82864E-01	1.00000E 10
	-1.30085E-02	1.82864E-01	1.00000E 10
	-4.20469E-01	-1.04041E 00	1.00000E 10
	-4.20469E-01	1.04041E 00	1.00000E 10

STABILITY CHARACTERISTICS	PHUGOID MODE	SHORT PERIOD
OMEGA N=	.1833	1.1222
ZETA=	.0710	.3747
PERIOD=	34.3599	6.0391
TIME TO .5=	53.2842	1.6485
TIME TO .1=	177.0064	5.4762

FIGURE 35 (CONT)

FORCING FUNCTIONS

U/T NUMERATOR - K(S**3 + A*S**2 + B*S + C)

K= 1.46229E-03	A= 8.06837E-01	B= 1.37939E 00	C= 2.00330E-02
ROOTS	REAL PART	IMAGINARY PART	CONVERGENCE
	-1.46463E-02	0	1.00000E 10
	-3.96095E-01	-1.10041E 00	1.00000E 10
	-3.96095E-01	1.10041E 00	1.00000E 10

ALPHA/T NUMERATOR - K(S**2 + A*S + B)

K= -6.16E-06	A= 7.75070E-02	B= 2.66899E-02	
ROOTS	REAL PART	IMAGINARY PART	CONVERGENCE
	-3.87535E-02	-1.58707E-01	1.00000E 10
	-3.87535E-02	1.58707E-01	1.00000E 10

U/ETA NUMERATOR - K(S**2 + A*S + B)

K= 2.73E-01	A= 2.89130E 02	B= 1.01675E 02	
ROOTS	REAL PART	IMAGINARY PART	CONVERGENCE
	-3.52088E-01	0	1.00000E 10
	-2.88778E 02	0	1.00000E 10

ALPHA/ETA NUMERATOR - K(S**3 + A*S**2 + B*S + C)

K=-8.22432E-02	A= 2.74956E 01	B= 1.11869E 00	C=-7.08027E 01
ROOTS	REAL PART	IMAGINARY PART	CONVERGENCE
	-2.73601E 01	0	1.00000E 10
	1.54236E 00	0	1.00000E 10
	-1.67783E 00	0	1.00000E 10

FIGURE 35 (CONT)

```
                                                                     05 02 66
      PROGRAM AEROFUN
C         *****
C     THIS PROGRAM IS DESIGNED TO COMPUTE THE DIMENSIONAL AERODYNAMIC
C     DERIVATIVES FROM BASIC WIND TUNNEL DATA AND AIRCRAFT DIMENSIONS
C         *****
      READ 10,XLAMBDA,CLA,CLAT,TR,AR,CH,SW,HT,ET,ST,TAU,XAC,XCG,XL,
     1XLB,SB,DEDAI,A,B
   10 FORMAT(7F10.0/7F10.0/5F10.0)
      READ 1,B,D,ZE
    1 FORMAT(3F10.0)
      READ 2,U,RHO,W,THETAD,ALPHA,CL,CLU,CD,CDA,CDU,CDQ,CDDE,CMA,CT
    2 FORMAT(7F10.0/7F10.0)
      CALL COMPTDER(XLAMBDA,CLA,CLAT,TR,AR,CH,SW,HT,ET,ST,TAU,A,B,XAC,
     1XCG,XL,XLB,SB,DEDAI,CLQ,CLDE,CMAD,CMQ,CMDE,CMDD,DEDA)
      PRINT 6
    6 FORMAT(1H1)
      PRINT 15,CLQ,CLDE,CMAD,CMQ,CMDE,CMDD,DEDA
   15 FORMAT(//25X36H COMPUTED NONDIMENSIONAL DERIVATIVES///5X4HCLQ=
     1E12.3,10H 1/RAD/SEC, 5X5HCLDE=E12.3,6H 1/RAD,5X5HCMAD=E12.3,
     211H 1/RAD/SEC2,//5X4HCMQ=E12.3,10H 1/RAD/SEC, 5X5HCMDE=E12.3,
     36H 1/RAD,5X7HCMDDOT=E12.3,11H 1/RAD-SEC2,// 5X5HDEDA=E12.3,
     48H RAD/RAD///)
      CALL ENGDIMEN(U,RHO,W,SW,B,CH,ZE,D,XL,CL,CLA,CLU,CLQ,CLDE,CD,CDA,
     1CDU,CDQ,CDDE,CMA,CMAD,CMQ,CMDE,CT,THETAD,XU,XW,XQ,XETA,XT,ZU,ZW,
     2ZQ,ZET,ZT,XMU,XMW,XMWD,XMQ,XMETA,XMT,GCTHETA,GSTHETA)
      PRINT 5,U,RHO,W,ALPHA,THETAD
    5 FORMAT(//35X,18H FLIGHT CONDITIONS// 5X6HUSUBO= F5.0,7H FT/SEC,7X
     19H DENSITY=E13.5,12H SLUGS/CB FT,5X8H WEIGHT=F7.0,4H LBS//
     2 5X6HALPHA=,F6.2,4H DEG,10X6HTHETA=,F6.2,4H DEG,///)
      PRINT 7
    7 FORMAT(///28X44H ENGLISH DIMENSIONAL AERODYNAMIC DERIVATIVES//)
      PRINT 8,XU,ZU,XMU,XW,ZW,XMW,GCTHETA,GSTHETA,XMWD,XQ,ZQ,XMQ,XT,ZT,
     1XMT,XETA,ZET ,XMETA
    8 FORMAT( 5X3HXU=F7.3,6H 1/SEC,16X3HZU=F7.3,6H 1/SEC,16X3HMU=E12.3,
     19H 1/SEC-FT,// 5X3HXW=F7.3,6H 1/SEC,16X3HZW=F7.3,6H 1/SEC,16X
     23HMW=E12.3,9H 1/SEC-FT,// 5X,13HGCOS(THETAO)=F7.3,8H FT/SEC2,4X
     313HGSIN(THETAO)=F7.3,8H FT/SEC2,4X6HMWDOT=E12.3,5H 1/FT,// 5X
     43HXQ=E12.3,11H FT/SEC-RAD,6X3HZQ=E12.3,10H 1/SEC-RAD,7X3HMQ=F7.3,
     510H 1/SEC-RAD,// 5X,3HXT=E12.3,11H FT/SEC2-LB,6X3HZT=E12.3,
     611H FT/SEC2-LB,6X3HMT=E12.3,10H 1/SEC2-LB,// 5X,5HXETA=E12.3,
     712H FT/SEC2-RAD,3X5HZETA=F7.3,12H FT/SEC2-RAD,8X5HMETA=F7.3,
     811H 1/SEC2-RAD,///)
      END
```

FIGURE 35 (CONT)

COMPUTED NONDIMENSIONAL DERIVATIVES

CLQ= -2.583E 00 1/RAD/SEC	CLDE= 5.803E-01 1/RAD	CMAD= -7.830E-01 1/RAD/SEC2
CMQ= -4.245E 00 1/RAD/SEC	CMDE= -6.936E-01 1/RAD	CMDDOT= 6.190E 04 1/RAD-SEC2
DEDA= 4.722E-01 RAD/RAD		

FLIGHT CONDITIONS

USUBO= 234 FT/SEC	DENSITY= 2.37800E-03 SLUGS/CB FT	WEIGHT= 22000 LBS
ALPHA= 11.10 DEG	THETA= 8.10 DEG	

ENGLISH DIMENSIONAL AERODYNAMIC DERIVATIVES

XU= -.060 1/SEC	ZU= -.265 1/SEC	MU= 1.852E-04 1/SEC-FT
XW= -.014 1/SEC	ZW= -.427 1/SEC	MW= -4.865E-03 1/SEC-FT
GCOS(THETAO)= 31.853 FT/SEC2	GSIN(THETAO)= 4.533 FT/SEC2	MWDOT= -2.523E-04 1/FT
XQ= 0 FT/SEC-RAD	ZQ= -2.322E 00 1/SEC-RAD	MQ= -.320 1/SEC-RAD
XT= 1.462E-03 FT/SEC2-LB	ZT= -2.170E-05 FT/SEC2-LB	MT= -4.552E-06 1/SEC2-LB
XETA= -1.642E 00 FT/SEC2-RAD	ZETA=-20.720 FT/SEC2-RAD	META= -2.078 1/SEC2-RAD

FIGURE 35 (CONT)

```
      SUBROUTINE STAB(U0,THETAD,XU,XW,ZU,ZW,ZQ,XMU,XMW,XMWD,XMQ,C,U,V,
     1CONV,WNLP,WNSP,ZLP,ZSP,TLP,TSP,THLP,THSP,TTLP,TTSP)
C                 ****
C     THIS SUBPROGRAM COMPUTES THE COEFFICIENTS OF THE CHARACTERISTIC
C     EQUATION, THE ROOTS OF THE EQUATION AND THE CHARACTERISTICS
C     OF THE SHORT PERIOD AND LONG PERIOD MOTION
C                 ****
      DIMENSION C(5), U(4), V(4),CONV(4)
      PI=4.*ATANF(1.) $ THETA0=THETAD*PI/180.
      GCTHETA=32.174*COSF(THETA0)  $ GSTHETA=32.174*SINF(THETA0)
      ZQPLSU0=ZQ+U0
C COMPUTE COEFFICIENTS OF CHARACTERISTIC EQUATION
      N=4 $ NN= N+1 $ C(1)=1.
      C(2) = -XU -ZW -XMQ -XMWD*ZQPLSU0
      C(3)=XU*ZW+XMQ*(XU+ZW)-XW*ZU-XMW*ZQPLSU0+XMWD*(XU*ZQPLSU0 +
     1GSTHETA)
      C(4)=XMQ*(-XU*ZW+XW*ZU)+XMU*(GCTHETA-XW*ZQPLSU0)+XMWD*(ZU*
     1 GCTHETA -XU*GSTHETA)+XMW*(XU*ZQPLSU0+GSTHETA)
      C(5)=GCTHETA*(XMW*ZU-ZW*XMU)+GSTHETA*(XMU*XW-XU*XMW)
C FIND ROOTS OF CHARACTERISTIC EQUATION
      CALL RTPLSUB(N,C,U,V,CONV,IER)
      IF(ABSF(U(1)).GT.ABSF(U(3)))10,11
   10 N=3 $ M=1 $ GO TO 12
   11 N=1 $ M=3
   12 ULP=ABSF(U(N)) $ VLP=ABSF(V(N))
      USP=ABSF(U(M)) $ VSP=ABSF(V(M))
      WNSP=SQRTF(USP**2+VSP**2) $ ZSP=USP/WNSP
      WNLP=SQRTF(ULP**2+VLP**2) $ ZLP=ULP/WNLP
      TSP=(2.*PI)/VSP  $THSP=(ABSF(LOGF(.5)))/USP
      TTSP=ABSF(LOGF(.1)) /USP    $ TLP=(2.*PI)/VLP
      THLP=(ABSF(LOGF(.5)))/ULP   $TTLP=ABSF(LOGF(.1)) /ULP
      END
```

FIGURE 35 (CONT)

```
      SUBROUTINE FORCEFUN(U0,THETAD,XU,XW,XT,ZU,ZW,ZQ,ZNU,XMU,XMW,XMWD,
     1XMQ,XMT,XMNU)
C                   ****
C     THIS SUBPROGRAM COMPUTES AND PRINTS THE COEFFICIENTS AND THE
C     ROOTS OF THE POLYNOMINAL NUMERATORS OF THE TRANSFER FUNCTIONS
C     ALPHA/ETA, U/ETA, ALPHA/T, AND U/T
C                   ****
      DIMENSION C(5), U(4), V(4),CONV(4)
      PRINT 6
    6 FORMAT(1H1)
      PI=4.*ATANF(1.) $ THETA0=THETAD*PI/180. $ ZQPLSU0=ZQ+U0
      GCTHETA=32.174*COSF(THETA0)  $ GSTHETA=32.174*SINF(THETA0)
C COMPUTE COEFFICIENTS OF U/T NUMERATOR POLYNOMIAL
      N=3 $ NN=N+1  $ C(1)=1.0
      C(2)=-XMQ -ZW -XMWD*ZQPLSU0
      C(3)=XMQ*ZW-XMW*ZQPLSU0+XMWD*GSTHETA+(XMT/XT)*(XW*ZQPLSU0-
     1GCTHETA)
      C(4)=XMW*GSTHETA+(XMT/XT)*(ZW*GCTHETA-XW*GSTHETA)
C FIND ROOTS OF U/T NUMERATOR
      CALL RTPLSUB(N,C,U,V,CONV,IER)
      PRINT 64
   64 FORMAT(///,35X18H FORCING FUNCTIONS//)
      PRINT 57
   57 FORMAT(/   ,5X42HU/T NUMERATOR - K(S**3 + A*S**2 + B*S + C) /)
      PRINT 61,XT,(C(J),J=2,NN)
   61 FORMAT(5X2HK=E12.5,5X2HA=E12.5,5X2HB=E12.5,5X2HC=E12.5,/)
      PRINT 56
   56 FORMAT(12X5HROOTS,10X9HREAL PART,7X14HIMAGINARY PART,9X
     111HCONVERGENCE/)
      PRINT 55,(U(I),V(I),CONV(I),I=1,N)
   55 FORMAT(17X,3E20.5)
```

FIGURE 35 (CONT)

```
C COMPUTE COEFFICIENTS OF ALPHA/T NUMERATOR
      N=2 $ NN=N+1
      AT= XT*ZU + ZQPLSUO*XMT $ XK= AT/U0
      BT=XT*(XMU*ZQPLSUO-ZU*XMQ)-XMT*(XU*ZQPLSUO+GSTHETA)
      CT=GSTHETA*(XU*XMT-XMU*XT)-ZU*XMT*GCTHETA
      C(2)=BT/AT $ C(3) = CT/AT
C FIND ROOTS
      CALL RTPLSUB(N,C,U,V,CONV,IER)
      PRINT 58
   58 FORMAT(/    ,5X37HALPHA/T NUMERATOR - K(S**2 + A*S + B) /)
      PRINT 60,XK,C(2),C(3)
   60 FORMAT(5X2HK=E12.2,5X2HA=E12.5,5X2HB=E12.5,/)
      PRINT 56
      PRINT 55,(U(I),V(I),CONV(I),I=1,N)
C COMPUTE COEFFICIENTS OF U/ETA NUMERATOR
      XK=ZNU*XW
      C(2)=(XMNU/ZNU)*ZQPLSUO-(GCTHETA/(ZNU*XW))*(XMNU+XMWD*ZNU)+XMQ
      C(3)=(GCTHETA/(ZNU*XW))*(XMNU*ZW-XMW*ZNU)+GSTHETA*XMNU/ZNU
C FIND ROOTS
      CALL RTPLSUB(N,C,U,V,CONV,IER)
      PRINT 59
   59 FORMAT(/    ,5X35HU/ETA NUMERATOR - K(S**2 + A*S + B)/)
      PRINT 60,XK,C(2),C(3)
      PRINT 56
      PRINT 55,(U(I),V(I),CONV(I),I=1,N)
C COMPUTE COEFFICIENTS OF ALPHA/ETA NUMERATOR
      N=3 $ NN=N+1 $ XK=ZNU/U0
      C(2)=(XMNU/ZNU)*ZQPLSUO-XMQ-XU
      C(3)=XU*XMQ-(XMNU/ZNU)*(GSTHETA+XU*ZQPLSUO)
      C(4)=XMNU*GCTHETA+(XMNU/ZNU)*(XU*GSTHETA-ZU*GCTHETA)
```

FIGURE 35 (CONT)

```
C FIND ROOTS
      CALL RTPLSUB(N,C,U,V,CONV,IER)
      PRINT 62
   62 FORMAT(/    ,5X48HALPHA/ETA NUMERATOR - K(S**3 + A*S**2 + B*S + C)/
     1  )
      PRINT 61,XK,(C(J),J=2,NN)
      PRINT 56
      PRINT 55,(U(I),V(I),CONV(I),I=1,N)
      END


      SUBROUTINE ENGDIMEN(U,R,W,S,B,C,ZE,D,XL,CL,CLA,CLU,CLQ,CLDE ,CD,
     1CDA,CDU,CDQ,CDDE ,CMA,CMAD,CMQ,CMDE ,CT,THETAD,XU,XW,XQ,XETA,
     2XT,ZU,ZW,ZQ,ZET,ZT,XMU,XMW,XMWD,XMQ,XMETA,XMT,GCTHETA,GSTHETA)
C              ****
C     THIS SUBRROGRAM COMPUTES THE ENGLISH DIMENSIONAL DERIVATIVES
C     FROM THE NACA NONDIMENSIONAL DERIVATIVES
C              ****
      PI=4.*ATANF(1.) $ THETAO=THETAD*PI/180.  $ ZER=ZE*PI/180.
      XM=W/32.174 $ BA=R*S*U/XM  $ BB=BA/2.  $ BC=BA*XL  $ BD=BB*U
      BE=R*S*U/B  $BF=BE*(C/2.)**2  $ ZQ=BA*C/4.*CLQ
      XMW=BE*(C/2.)*CMA  $ XMWD=(BF/U)*CMAD  $ XMQ=BF*CMQ $ XMT=-D/B
      XU=BA*(-CD-CDU) $ XW=BB*(CL-CDA) $ XQ=BC*CDQ $ XETA=BD*( CDDE)
      XT=COSF(ZER)/XM  $ ZU=BA*(-CL-CLU) $ ZW=BB*(-CLA-CD)
      ZET=BD*(-CLDE)   $ZT=   -SINF(ZER)/XM  $XMU= BE*(CT*D)
      XMETA=(BE*U*C/2.)*CMDE  $ GCTHETA=32.174*COSF(THETAO)
      GSTHETA=32.174*SINF(THETAO)
      END
```

FIGURE 35 (CONT)

```
      SUBROUTINE COMPTDER(XLAMBDA,CLA,CLAT,TR,AR,C,SW,HT,ET,ST,TAU,A,B,
     1XAC,XCG,XL,XLB,SB,DEDAI,CLQ,CLDE,CMAD,CMQ,CMDE,CMDD,DEDA)
C                 ****
C     THIS SUBPROGRAM COMPUTES THOSE AERODYNAMIC DERIVATIVES WHICH
C     MUST NORMALLY BE APPROXIMATED FROM WIND TUNNEL DATA
C     CL Q, CL DELTA E, CM ALPHA DOT, CM Q, CM DELTA E, CM DELTA DOT,
C     D EPSILON D ALPHA
C                 ****
      PI=4.*ATANF(1.)$RLAM=XLAMBDA*PI/180.
      CLAM=COSF(RLAM)  $TLAM=TANF(RLAM)
      VT=ET*ST/SW   $XBAR=XAC-XCG   $XLOVC=XL/C
      IF(DEDAI)2,1,2
    1 DEDA=((27.*CLA*CLAM**1.66)/(( .525+.475*TR**.736)*(AR**.725)
     1*(XLOVC**.2)*EXPF(.385*HT/C)))*PI/180.
      GO TO 3
    2 DEDA=DEDAI
    3 CLQW=-CLA*(.44+2.*XBAR) $CLQT=-2.*CLAT*VT*XLOVC  $CLQ=CLQW+CLQT
      CLDE= CLAT*VT*TAU   $CMAD=-2.*CLAT*VT*DEDA*XLOVC**2
      CMQW=-CLQW*XBAR -CLA*((AR+2.*CLAM)/(8.*AR)+(((AR*TLAM)**2)/24.)
     1*((AR+2.*CLAM)/(AR+6.*CLAM)))
      CMQT= CLQT*XLOVC  $CMQB=4.*(XLOVC**2)*(-.01)*(SB/SW)*((XLB/XL)**2)
      CMQ=CMQW+CMQT+CMQB  $CMDE=-CLAT*VT*XLOVC*TAU
      CMDD=(ST/SW)*(A+CLAT*B)
      END
```

characteristic equation is the expansion of the main determinant as shown below.

$$CH = \begin{vmatrix} S - X_u & -U_o X_w & g\cos\Theta_o \\ -\dfrac{Z_u}{U_o} & S - Z_w & -\left(\dfrac{Z_q}{U_o} + 1\right) S + \dfrac{g \sin\Theta_o}{U_o} \\ -M_u & -U_o(M_w - S M_{\dot{w}}) & S^2 - M_q S \end{vmatrix}$$

$$CH = S^4 + A S^3 + B S^2 + C S + D$$

$$A = -X_u - Z_w - M_q - M_{\dot{w}}(Z_q + U_o)$$

$$B = X_u Z_w + M_q(X_u + Z_w) - X_w Z_u - M_w(Z_q + U_o) + M_{\dot{w}}[X_u(Z_q + U_o) + g \sin\Theta_o]$$

$$C = M_q(-X_u Z_w + X_w Z_u) + M_u[g\cos\Theta_o - X_w(U_o + Z_q)] + M_{\dot{w}}(Z_u g \cos\Theta_o - X_u g \sin\Theta_o) + M_w[X_u(Z_q + U_o) + g \sin\Theta_o]$$

$$D = g\cos\Theta_o(M_w Z_u - Z_w M_u) + g \sin\Theta_o(M_u X_w - X_u M_w)$$

The transfer function relating airspeed to a thrust input perturbation is then:

$$\frac{u}{T} = \frac{\begin{vmatrix} X_T & -U_o X_w & g\cos\Theta_o \\ 0 & S - Z_w & -\left(\dfrac{Z_q}{U_o} + 1\right) S + \dfrac{g \sin\Theta_o}{U_o} \\ M_T & -U_o(M_w - S M_{\dot{w}}) & S^2 - M_q S \end{vmatrix}}{CH}$$

$$\frac{u}{T} = \frac{X_T\left(S^3 + A_{uT} S^2 + B_{uT} S + C_{uT}\right)}{CH}$$

$$A_{uT} = -M_q - Z_w - M_{\dot{w}}\left(Z_q + U_o\right)$$

$$B_{uT} = -M_q Z_w - M_w\left(Z_q + U_o\right) + M_{\dot{w}}\, g \sin\Theta_o + \frac{M_T}{X_T}\left[X_w\left(Z_q + U_o\right) - g\cos\Theta_o\right]$$

$$C_{uT} = M_w g \sin\Theta_o + \frac{M_T}{X_T}\left[Z_w g \cos\Theta_o - X_w g \sin\Theta_o\right]$$

The other transfer functions follow in a like manner.

$$\frac{\alpha}{T} = \frac{\begin{vmatrix} S - X_u & X_T & g\cos\Theta_o \\ -\dfrac{Z_u}{U_o} & 0 & -\left(\dfrac{Z_q}{U_o} + 1\right)S + \dfrac{g\sin\Theta_o}{U_o} \\ M_u & M_T & S^2 - M_q S \end{vmatrix}}{CH}$$

$$\frac{\alpha}{T} = \frac{A_T S^2 + B_T S + C_T}{U_o(CH)} = K\left(S^2 + C_1 S + C_2\right)$$

$$K = \frac{A_T}{U_o} \qquad C_1 = \frac{B_T}{A_T} \qquad C_2 = \frac{C_T}{A_T}$$

$$A_T = X_T Z_u + M_T(Z_q + U_0)$$

$$B_T = [M_u(Z_q + U_0) - Z_u M_q] X_T - M_T[X_u(Z_q + U_0) + g \sin \Theta_0]$$

$$C_T = g \sin \Theta_0 (X_u M_T - M_u X_T) - Z_u M_T g \cos \Theta_0$$

$$\frac{u}{\eta} = \frac{\begin{vmatrix} 0 & -U_0 X_w & g \cos \Theta_0 \\ \dfrac{Z_\eta}{U_0} & S - Z_w & -\left(\dfrac{Z_q}{U_0} + 1\right) S + \dfrac{g \sin \Theta_0}{U_0} \\ M_\eta & -U_0(M_w + S M_{\dot{w}}) & S^2 + M_q S \end{vmatrix}}{CH}$$

$$\frac{u}{\eta} = \frac{Z_\eta X_w (S^2 + A_{u\eta} S + B_{u\eta})}{CH}$$

$$A_{u\eta} = \frac{M_\eta}{Z_\eta}\left(Z_q + U_0\right) - \frac{g \cos \Theta_0}{Z_\eta X_w}\left(M_\eta + M_{\dot{w}} Z_\eta\right) + M_q$$

$$B_{u\eta} = \frac{g \cos \Theta_0}{Z_\eta X_w}\left(M_\eta Z_w - M_w Z_\eta\right) + \frac{M_\eta}{Z_\eta} g \sin \Theta_0$$

$$\frac{\alpha}{\eta} = \frac{\begin{vmatrix} S - X_u & 0 & g\cos\Theta_o \\ -\dfrac{Z_u}{U_o} & \dfrac{Z_\eta}{U_o} & -\left(\dfrac{Z_q}{U_o}+1\right)S + \dfrac{g\,\mathrm{SIN}\,\Theta_o}{U_o} \\ -M_u & M_\eta & S^2 - M_q S \end{vmatrix}}{CH}$$

$$\frac{\alpha}{\eta} = \frac{Z_\eta\left(S^3 + A_{\alpha\eta}S^2 + B_{\alpha\eta}S + C_{\alpha\eta}\right)}{U_o(CH)}$$

$$A_{\alpha\eta} = \frac{M_\eta}{Z_\eta}\left(Z_q + U_o\right) - M_q - X_u$$

$$B_{\alpha\eta} = X_u M_q - \frac{M_\eta}{Z_\eta}\left[g\,\mathrm{SIN}\,\Theta_o + X_u\left(Z_q + U_o\right)\right]$$

$$C_{\alpha\eta} = M_\eta\, g\cos\Theta_o + \frac{M_\eta}{Z_\eta}\left[X_u\, g\,\mathrm{SIN}\,\Theta_o - Z_u\, g\cos\Theta_o\right]$$

7.3 Evaluation of the Closed Loop Transfer Function.

The closed loop transfer function was derived as Eq. (2-8)

$$\frac{u}{u_c} = \frac{\frac{T_c}{u_c}\,\frac{T_f}{T_c}\,\frac{T}{T_f}\,\frac{u}{T}}{1 + D_u\,\frac{T_f}{u_c}\,\frac{T_f}{T_\epsilon}\,\frac{T}{T_f}\,\frac{u}{T} - D_\alpha\,\frac{T_c}{\alpha_e}\,\frac{T_f}{T_\epsilon}\,\frac{T}{T_f}\,\frac{\alpha}{T}}$$

If all delays are assumed to negligible, this reduces to

$$\frac{u}{u_c} = \frac{\frac{T_c}{u_c}\,\frac{u}{T}}{1 + \frac{T_c}{u_\epsilon}\,\frac{u}{T} - \frac{T_c}{\alpha_e}\,\frac{\alpha}{T}}$$

Substituting the transfer functions defined in Table IV with

$$\frac{u}{T} = \frac{UN}{CH} \qquad \frac{\alpha}{T} = \frac{\alpha N}{CH}$$

gives

$$\frac{u}{u_c} = \frac{\frac{K_u(S + k_u)}{S}\,\frac{UN}{CH}}{1 + \frac{K_u(S + k_u)}{S}\,\frac{UN}{CH} - K_\alpha\,\frac{\alpha N}{CH}}$$

therefore

$$\frac{u}{u_c} = \frac{K_u(S + k_u)(UN)}{S(CH) + K_u(S + k_u)(UN) - K_\alpha\,(\alpha N)}$$

The denominator of this expression is a 5th order polynomial shown below.

$S^5 + C_1S^4 + C_2S^3 + C_3S^2 + C_4S + C_5$ where, using the same notation as 7.2

$$C_1 = A + K_uX_T$$

$$C_2 = B + K_uX_TA_{uT} + K_uk_uX_T - K_\alpha A_{\alpha T}$$

$$C_3 = C + K_uX_TB_{uT} + K_uk_uX_TA_{uT} - K_\alpha A_{\alpha T}B_{\alpha T}$$

$$C_4 = D + K_u X_T C_{uT} + K_u k_u X_T B_{uT} - K_\alpha A_{\alpha T} C_{\alpha T}$$

$$C_5 = K_u k_u X_T C_{uT}$$

Then

$$\frac{u}{u_c} = \frac{X_T K_u (S + k_u)(S^3 + A_{uT} S^2 + B_{uT} S + C_{uT})}{S^5 + C_1 S^4 + C_2 S^3 + C_3 S^2 + C_4 S + C_5}$$

From this expression the root locus was obtained by digital computer means. The integral gain produces one variable zero. The other zeroes of the function are invariant with changes of the gains. The root locus plots obtained are shown in Figures 33 and 34. The most desirable gains can now be chosen from these plots using standard pole-zero methods. [2] However, this type of an analysis would have to be made for each loop involved to arrive at the best combination of gains for both the airspeed and the angle of attack response to gusts.

7.4 Analogue Simulation

Substituting the aerodynamic values into Eqs 2-4 gives

$$\dot{u} = -.060u - 3.32\alpha - 31.85\theta + .00145T - 1.64\eta$$

$$\dot{\alpha} = -.00113u - .4265\alpha + \dot{\theta} - .01935\theta - .0815\eta$$

$$\ddot{\theta} = 1.85\times10^{-4}u - 1.14\alpha - .0415\dot{\alpha} - .338\dot{\theta} - 4.55\times10^{-6}T - 2.25\eta$$

These equations were then scaled using the relations defined in Table VIII. Real time was used. Using this scaling the voltage form of the equations becomes

$$\dot{\bar{u}} = -.101\bar{u} - .0116\bar{\alpha} - .111\bar{\theta} + .292\bar{T} - .006\bar{\eta}$$

$$\dot{\bar{\alpha}} = -.544\bar{u} - .4265\bar{\alpha} + \dot{\bar{\theta}} - .01935\bar{\theta} - .0815\bar{\eta}$$

$$\ddot{\bar{\theta}} = .0311\bar{u} - .398\bar{\alpha} - .0145\dot{\bar{\alpha}} - .1181\dot{\bar{\theta}} - .785\bar{\eta}$$
$$- .091\bar{T}$$

The power compensator equation was scaled as (Eq 2-5 without delays)

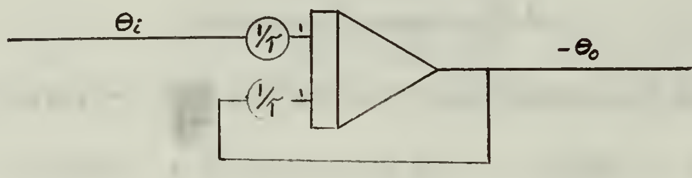
$$\overline{Tc} = \frac{K_{\alpha}}{5.73\text{x}10^4}\,\overline{\alpha}_e - \frac{K_u k_u}{118.9}\int\overline{u_e} - \frac{K_u}{118.9}\,\overline{u_e}$$

The delays were simulated by the general relationship shown below were θ_o is the delayed signal and θ_i is the input signal.

If $\dfrac{\theta_o}{\theta_i} = \dfrac{1}{1 + \tau S}$ then $\dot{\theta}_o = 1/\tau\theta_i - 1/\tau\theta_o$

or $\theta_o = \int 1/\tau\theta_i - 1/\tau\theta_o$

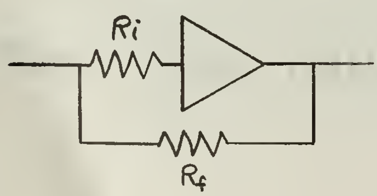
which gives the simulation circuit



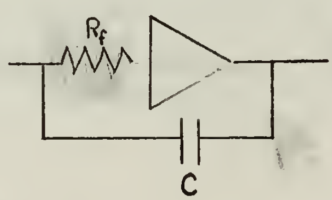
The values of $1/\tau$ can be obtained by either potentiomenter settings or by adjusting the input gains.

The schematic diagram for the wiring circuits is given in Figures 7 and 8. The number on each input is the gain of the input where the gain is as defined below:

For a summing amplifier



For an integrator



$$\text{Gain} = \frac{R_f}{R_i} \qquad\qquad \text{Gain} = \frac{1}{R_i C}$$

The potentiometer settings are given in Table IX. In order to keep the wiring diagram as simple as possible, feedback circuits are indicated by the use of numbered potentiometers or numbered units shown dashed. The squares indicate recorder channels. SW 1 and SW 2 are internal double throw-double pole switches. NC refers to the "normally closed" position of SW 2. Step gusts were simulated by means of initial conditions on the airspeed and angle of attack integrators. Sinusoidal gust inputs were obtained by means of a frequency oscillator.

TABLE VIII

SCALED VARIABLES

max voltage · 100
max out

Parameter	Maximum Value (per 100 volts)	Voltage Equation	Scale
$\dot{u}$	50 ft/sec^2	$\bar{\dot{u}} = 2u$	.5 ft/sec/volt
u, u_c	84.3 ft/sec 50 kts	$\bar{u} = 1.189u$ $\bar{u} = 2u$	.843 ft/sec/volt .5 kts/volt
$\alpha, \dot{\alpha}, \alpha_c$	.1745 rad 10 deg	$\bar{\alpha} = 573\,\alpha$ $\bar{\alpha} = 10\,\alpha$	.001745 rad/volt .1 deg/volt
$\ddot{\theta}$	.5 rad/sec^2	$\bar{\ddot{\theta}} = 200\,\ddot{\theta}$	.005 rad/sec^2/volt
$\dot{\theta}, \theta, \gamma, \eta$	.1745 rad 10 deg	$\bar{\dot{\theta}} = 573\,\theta$ $\bar{\theta} = 10\,\theta$	.001745 rad/volt .1 deg/volt
T	1000 lbs	$\bar{T} = T/100$	100 lb/volt

TABLE IX

POTENTIOMETER SETTINGS

Pot #	Setting	Input Parameter	Gain
1	.292	T	1
2	.006	η	1
3	.101	u	1
4	.0116	α	1
5	.111	θ	1
6	.101/.0116	u_g/α_g (Gusts)	1/1
8	.0815	η	1
9	.272	u	2
10	.2133	α	2
12	.0194	θ	1
13	.272/2133	u_g/α_g (Gusts)	2/2
14	.091	T	1
15	.393	η	2
16	.031	u	1
17	.199	α	2
18	.0145	$\dot{\alpha}$	1
21	.118	$\dot{\theta}$	1
22	.031/0	$u_g/\dot{\alpha}_g$ (Gusts)	1
23	Various	u_c	1
24	0/.199	u_g/α_g (Gusts)	2
27	K_u	u_e	10
28	K	α_e	1
29	$K_u k_u$	$\int u_e$	1

TABLE IX (CONT)

31&32	.5	T_c (Throttle Servo)	10
33&34	.435	T_f (Engine Delay)	2
35	Various	η	1
36	Various	η_c	1
38	.5	η delay	1

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13. ABSTRACT

A study of an Automatic Power Compensator with inputs of airspeed and angle of attack was conducted by means of a simplified analytical analysis and analogue simulation. Digital Computer programming was used for data reduction. The Chance Vought F-8 aircraft was used as the vehicle for the study. Loop gains were initially set by means of a simplified analysis on the independent phugoid and short period motions. The validity of such an analysis was demonstrated by analogue simulation of the complete system. The system is feasiable both for independent use or as a component of a complete automatic flight system.

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